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DECENTRALIZED STABILIZATION AND STRONG STABILIZATION OF A BICOPRIME FACTORIZED PLANT

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In this paper, a necessary and sufficient condition for decentralized stabilizability for expanding construction of large scale systems is established which involves the computation of blocking zeros and testing a rational function for sign changes at these blocking zeros. Results for the scalar as also multivariable cases are presented and a systematic procedure for designing the stabilizing controller is also outlined. The proposed theory is applicable to a wider class of systems than those for which existing methods can be used. There are a few matrix identities established in this paper which are of independent interest in Control Theory.

1. INTRODUCTION

The last two decades have seen a lot of activities in the field of large scale systems. One of the major sub-areas in the field is decentralized control of interconnected systems [4, 6, 7]. A large number of results have been obtained concerning stabilization and optimization. Historically, these results consider increasingly broader class of models and interconnections among them. However, the common assumption of all these problems is that there is a given "open-loop" whole interconnected system to be controlled. Usually, large scale systems are constructed from one subsystem by connecting other subsystems one after another. Moreover, the connection of a new subsystem is carried out while the existing interconnected system is in operation.

This work addresses the decentralized stabilization problem which is appropriate to such expanding construction of large scale systems [1]. In this problem (henceforth called the expanding system problem or ESP) the information structure constraint is the usual one, i. e., the local output feedback controllers can have access only to local outputs. For the expanding construction, at each stage of connection of a new subsystem, the local controllers in the already interconnected portion have to be such that they stabilize not only that portion but also the expanded system in cooperation with the controller for the newly connected subsystem. As it is impractical to tune local controllers every time a new subsystem is connected, the local controllers in the existing portion are not changed and the duty of stabilizing the expanded system is imposed on the local controller of the new subsystem. Obviously, this has to be designed using the information about the new subsystem, the already interconnected portion, and the interconnection between them.

The subsytems are assumed to be autonomous by themselves. So, it is presumed that a local controller has been implemented to the new subsystem before it is connected to the already existing portion and that the closed-loop new subsystem is stable. Now, the problem is that of picking up local controllers from the set of all stabilizing controllers of the new subsystem. The most natural paradigm of controller design in this context is the factorization approach [2, 9] since it gives the full set of stabilizing controllers in terms of a free parameter for any system. Finally, the problem is that of choosing an appropriate free parameter such that the expanded system is stable after connection.

For the expanding system problem, introduced by Davison and Ozgüner [1], Tan and Ikeda [8] presented a sufficient condition for decentralized stabilizability using factorization approach. However, the complete necessary and sufficient condition for the problem is still unknown. This means that the impossibility to solve the two equaions needed to check stabilizability renders the whole exercise of finding stabilizing controllers futile. Using the necessary and sufficient condition presented in this work, one can characterize the full class of such compensators. It is shown in this paper that the question of decentralized stabilizability of the expanding system problem is equivalent to the problem of division in a Proper Euclidean Domain with a minimum degree remainder. From the pioneering work of Youla et al [10], later rigourously formalized by Vidyasagar [9], it is known that this division problem is connected with the ability to solve a unit interpolation problem in a Proper Euclidean Domain. Their result is applied in this paper to find the necessary and sufficient conditions for decentralized stabilizability. It is shown that to check the existence of such a controller, one has only to check for any sign changes in a proper stable transfer function at the blocking zeros of a transfer matrix in $M(R_{ps})$. Consequently, the design of the controller amounts to solving the unit interpolation problem using a generalized Youla-like interpolation algorithm. Thus it is shown for the first time that the full set of decentrally stabilizing controllers that solves ESP are characterized by the set of interpolating units solving the afore-mentioned interpolation problem. It is made clear that this set of interpolating units constitutes an appropriate subset of free parameters from the set of all stabilizing controllers of the new subsystem.

In Tan and Ikeda [8], the class of subsystems is restricted to those which do not have any direct transmissions from the inputs to the outputs. In particular, it was pointed out that the generalization to handle a broader class of systems that allow direct transmissions from the interconnection inputs to the interconnection outputs is nontrivial. The necessary and sufficient condition established in this paper can take care of direct transmissions between all inputs and all outputs.

The paper is arranged in the following manner. An outline of the system model under investigation along with a pivotal theorem is included in Section 2. In Section 3, the scalar case is considered where the relevant theorem is presented. It also contains a detailed procedure for constructing the controller and an example as an application. Section 4 presents the Main Theorem and a few Lemmas which are required in the proof of it. All the Lemmas and Theorems are proved in this section. Section 5 summarizes the conclusions and points out the future directions of research. Finally, the notations are listed in Section 6 and the references in Section 7.

2. THE MODEL

Let us consider N number of subsystems S_i :

$$\begin{aligned} \dot{x}_i &= A_i x_i + B_i u_i + G_i v_i \\ y_i &= C_i x_i + F_{4i} u_i + F_{3i} v_i \\ w_i &= H_i x_i + F_{2i} u_i + F_{1i} v_i \quad i = 1, 2, \ldots, N \end{aligned} \tag{1}$$

with local controllers described by LC_i :

$$\begin{aligned} \dot{z}_i &= M_i z_i + L_i y_i \\ u_i &= J_i z_i + K_i y_i \qquad i = 1, 2, \ldots, N. \end{aligned} \tag{2}$$

Here, x_i is the state, u_i is the control input, y_i is the measured output, v_i is the interconnection input and w_i is the interconnection output of the ith subsystem S_i. Also z_i is the state of the local controller LC_i and A_i, B_i, C_i, H_i, G_i, M_i, L_i, J_i and K_i are all constant matrices which are of appropriate dimensions. It is also assumed that (A_i, B_i) are stabilizable and (C_i, A_i) are detectable.

The closed loop N number of subsystems are given by S_i^c :

$$\begin{aligned} \begin{bmatrix} \dot{x}_i \\ \dot{z}_i \end{bmatrix} &= \begin{bmatrix} A_i + B_i K_i C_i & B_i J_i \\ L_i C_i & M_i \end{bmatrix} \begin{bmatrix} x_i \\ z_i \end{bmatrix} + \begin{bmatrix} G_i \\ 0 \end{bmatrix} v_i \\ w_i &= [H_i \;\; 0] \begin{bmatrix} x_i \\ z_i \end{bmatrix} \qquad i = 1, 2, \ldots, N. \end{aligned} \tag{3}$$

In the 'Expanding System Problem', there is already an interconnected system composed of $N-1$ subsystems, each of which is modelled as S_i above. The model of the interconnected system is given by $\bar{S}_{N-1}^c$:

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} A + BKC + GEH & BJ \\ LC & M \end{bmatrix}$$

where x, y, u, v, z are of the form $x = [x_1^T \; x_2^T \; \cdots x_{N-1}^T]^T$ and A, B, C, G, H, J, K, L and M are all of the form $A =$ block diag$[A_1, \; A_2, \; \ldots, A_{N-1}]$. The matrix E represents the interconnection between v and w, i. e., $v = Ew$ where

$$E = \begin{bmatrix} 0 & E_{1,2} & E_{1,3} & \cdots & E_{1,N-1} \\ E_{2,1} & 0 & E_{2,3} & \cdots & E_{2,N-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ E_{N-1,1} & E_{N-1,2} & E_{N-1,3} & \cdots & 0 \end{bmatrix} \tag{4}$$

with $v_i = \sum_{j=1, j\neq i}^{N} E_{i,j} w_j$.

Now a new subsystem S_N^c is connected to the already interconnected system $\bar{S}_{N-1}^c$ and thereby we get the expanded system $\bar{S}_N^c$. The problem is to design the controller for the new subsystem S_N so that both the closed loop new subsystem S_N^c as well as the expanded system $\bar{S}_N^c$ become stable.

Tan and Ikeda [8] found the most general result available to date regarding the condition for such decentralized stabilizability of expanding systems. Their condition is valid for any value of interconnection gains and does not depend on the pattern and/or strength of it. To prove their point, they used the factorization approach of controller design [1, 5]. Another remarkable feature is that all the conditions are in terms of the new subsystem's parameters only. The subsystem can be viewed as a 2-input 2-output multivariable system as follows.

$$\begin{bmatrix} w_N \\ y_N \end{bmatrix} = \begin{bmatrix} Z_{N11} & Z_{N12} \\ Z_{N21} & Z_{N22} \end{bmatrix} \begin{bmatrix} v_N \\ u_N \end{bmatrix} \tag{5}$$

where using the $[A, B, C, D]$ data structure notation of [3]

$$\begin{aligned} Z_{N11} &= [A_N, G_N, H_N, 0] \\ Z_{N12} &= [A_N, B_N, H_N, 0] \\ Z_{N21} &= [A_N, G_N, C_N, 0] \\ Z_{N22} &= [A_N, B_N, C_N, 0]. \end{aligned} \tag{6}$$

To find the set of all stabilizing controllers, Z_{N22} can be factorized as

$$Z_{N22} = N_N D_N^{-1} = \tilde{D}_N^{-1} \tilde{N}_N \tag{7}$$

where $N_N, D_N, \tilde{N}_N$ and $\tilde{D}_N \in M(R_{ps}(s))$ so that they satisfy the *Aryabhatta identity*

$$\begin{bmatrix} Q_N & P_N \\ -\tilde{N}_N & \tilde{D}_N \end{bmatrix} \begin{bmatrix} D_N & -\tilde{P}_N \\ N_N & \tilde{Q}_N \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \tag{8}$$

for some P_N, Q_N, $\tilde{P}_N$ and $\tilde{Q}_N \in M(R_{ps})$.

The set of all stabilizing controllers for the subsystem S_N in (2) is given by

$$u_N = -(\tilde{P}_N + D_N R_N)(\tilde{Q}_N - N_N R_N)^{-1} y_N \tag{9}$$

where $R_N \in M(R_{ps})$ is the free parameter. With a specified R_N, (2) becomes the time-domain realization of the controller. The closed loop Nth subsystem has the following transfer function matrix $T_N(R_N)$ from input v_N to output w_N, which can be seen to be affine in R_N.

$$T_N(R_N) = T_{N1} - T_{N2} R_N T_{N3} \tag{10}$$

where

$$\begin{aligned} T_{N1} &= Z_{N11} - Z_{N12} P_N D_N Z_{N21} \\ T_{N2} &= Z_{N12} D_N \\ T_{N3} &= \tilde{D}_N Z_{N21}. \end{aligned}$$

The feedback structure of an expanded system is shown in Figure 1 where $\bar{T}_{N-1}$ is the transfer matrix of $\bar{S}^c_{N-1}$ from the interconnection input $\bar{v}_{N-1}$ to the interconnection output $\bar{w}_{N-1}$. The Nth subsystem S^c_N is connected to the existing system $\bar{S}^c_{N-1}$ through

$$\begin{aligned} \bar{v}_{N-1} &= \bar{E}_{N-1,N} w_N \\ v_N &= \bar{E}_{N,N-1} \bar{w}_{N-1} \end{aligned}$$

where

$$\begin{aligned} \bar{E}_{N-1,N} &= [E^T_{1,N} \ E^T_{2,N} \ \cdots E^T_{N-1,N}] \\ \bar{E}_{N,N-1} &= [E_{N,1} \ E_{N,2} \ \cdots E_{N,N-1}] \end{aligned}$$

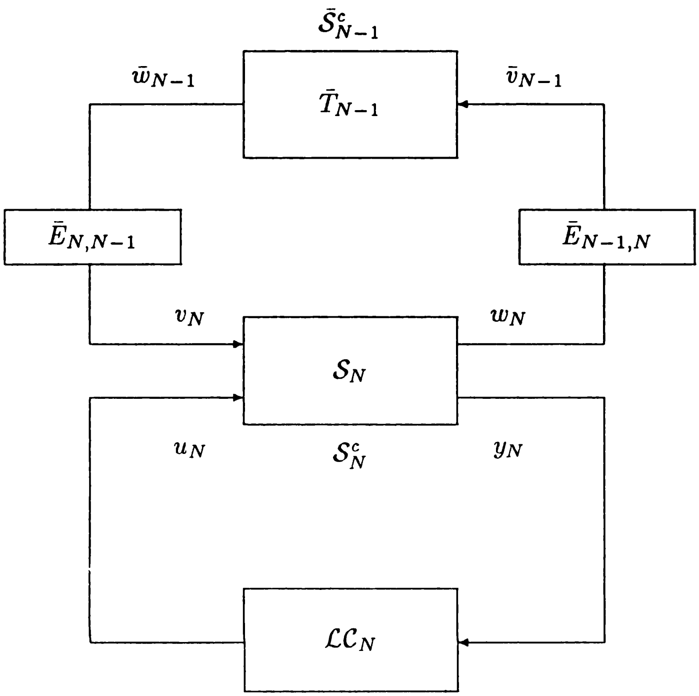

Fig. 1. Feedback structure of an expanded system.

The following Theorem ensures the decentralized stabilizability of the expanding system problem which directly follows from the results given in [8].

Theorem 2.1. Let LC_N be the set of all stabilizing controllers for the new closed loop subsystem S_N^c. A controller in LC_N will stabilize the expanded system $\bar{S}_N^c$ iff the the following condition is valid.

$$\min_{R_N \in M(R_{ps})} \delta(I - \bar{\psi}_{N-1} T_{N1} + \bar{\psi}_{N-1} T_{N2} R_N T_{N3}) = 0 \tag{11}$$

where $\bar{\psi}_{N-1} := \bar{E}_{N,N-1} \bar{T}_{N-1} \bar{E}_{N-1,N}$ and $\delta(t)$:= degree of $t \in M(R_{ps}(s))$ = number of right half plane zeros of t including ∞. This degree function makes $M(R_{ps}(s))$ a Proper Euclidean Domain.

Remark. For direct transmissions, y_i and w_i are changed to $y_i = C_i x_i + F_{4i} u_i + F_{3i} v_i$ and in (3) $A_i + B_i K_i C_i$, $B_i J_i$, $L_i C_i$, M_i, G_i and O are replaced by $A_i + B_i K_i \Xi_i C_i$, $B_i J_i + B_i K_i \Xi_i F_{4i} J_i$, $L_i \Xi_i C_i$, $M_i + L_i \Xi_i F_{4i} J_i$, $G_i + B_i K_i \Xi_i F_{3i}$, $L_i \Xi_i F_{3i}$ respectively, and in (6) Z_{N11}, Z_{N12}, Z_{N21} and Z_{N22} are modified to

$$\begin{aligned} Z_{N11} &= [A_N, G_N, H_N, F_{1N}] \\ Z_{N12} &= [A_N, B_N, H_N, F_{2N}] \\ Z_{N21} &= [A_N, G_N, C_N, F_{3N}] \\ Z_{N22} &= [A_N, B_N, C_N, F_{4N}]. \end{aligned}$$

The expression for w_i in (3) is changed to $w_i = [H_i + F_{2i} K_i \Xi_i C_i] x_i + F_{2i} [J_i + K_i \Xi_i F_{4i} J_i] z_i + [F_{1i} + F_{2i} K_i \Xi_i F_{3i}] v_i$ where $\Xi_i = (I - F_{4i} K_i)^{-1}$ is assumed to exist, i.e., $(I - F_{4i} K_i)$ is nonsingular for well-posedness.

3. SCALAR CASE

3.1. Theory

The Main Theorem for the Scalar case is given below.

Theorem 3.1. Let $\sigma_1, \sigma_2, \ldots, \sigma_l$ be the real nonnegative distinct zeros including ∞ of $\bar{\psi}_{N-1} T_{N2} T_{N3}$ arranged in the ascending order. Then the following statements are equivalent.

–

$$\min_{R_N \in R_s(s)} \delta(1 - \bar{\psi}_{N-1} T_{N1} + \bar{\psi}_{N-1} T_{N2} R_N T_{N3}) = 0. \tag{12}$$

– The following real numbers have the same sign.

$$\{f(\sigma_1), f(\sigma_2), \cdots, f(\sigma_l)\}$$

where

$$f := 1 - \bar{\psi}_{N-1} T_{N1}.$$

Once the existence of the controller is established with the help of Theorem 3.1, the free parameter $R_N \in R_{ps}$ is chosen to make equation (12) valid. The procedure for finding such an R_N is algebraically equivalent to dividing p by q where p and q are any two elements in the Proper Euclidean Domain R_{ps},with a minimum (in this case, zero) degree remainder. We define two functions $f, g \in R_{ps}(s)$ as follows.

$$f \;:=\; 1 - \bar{\psi}_{N-1}T_{N1} \tag{13}$$
$$g \;:=\; \bar{\psi}_{N-1}T_{N2}T_{N2}. \tag{14}$$

Note that in both the function definitions, each element on the right hand sides is an element of R_{ps}. Now the problem is to find an $R_N \in R_{ps}$ such that

$$\min_{R_N \in R_{ps}} \delta(f + gR_N) = 0. \tag{15}$$

Suppose that $\sigma_1, \sigma_2, \ldots, \sigma_l$ denote the distinct nonnegative real zeros of the function g including ∞ in ascending order and let $s_{l+1}, \ldots, s_n$ be distinct complex zeros of g with positive imaginary parts. Let $m_1, m_2, \ldots, m_l$ and $m_{l+1}, \ldots, m_n$ be their corresponding multiplicities respectively. Moreover, let $\{r_{ij}\}$ be a set of numbers defined by

$$r_{ij} := f^{(j)}(s_i), \quad i = 1, 2, \ldots, n;\; j = 0, 1, \ldots, m_i - 1$$

where $s_i = \sigma_i, i = 1, 2, \ldots, l$. *It is to be noted that if $\sigma_l = \infty$, then r_{lj}'s are found after transforming it to z-domain.*

Remark. The parameterization of R_N, satisfying (15) is possible as given in [9]. Such parameterization is necessary for optimization of some performance criteria for tracking, disturbance rejection etc.

The unit interpolation algorithm given in [9] can be used for solving this problem. However, a minor modification is needed for handling zeros at infinity with multiplicity greater than one, as otherwise it leads to an improper R_N which is not in R_{ps}.

3.2. Examples

Example 1. This is an illustrative example of the case in which the sufficient conditions are not satisfied but there exists a local controller LC_N which stabilizes both the new subsystem as well as the expanded system (i. e., it satisfies the necessary and sufficient condition).

$$\bar{\psi}_{N-1} = 1;\; T_{N1} = -\frac{(s+3)}{(s+1)^2};\; T_{N2} = \frac{(s-1)}{(s+1)^2};\; T_{N3} = \frac{(s-2)}{(s+1)}.$$

It is clear that there does not exist any X_N and Y_N in R_s satisfying $T_{N2}X_N = T_{N1}$ and $Y_N T_{N3} = T_{N1}$ (see [8]). Now applying Theorem 3.1 with $f = 1 - \bar{\psi}_{N-1}T_{N1}$, it follows that the sequence $\{f(1), f(2), f(\infty)\} = \{2, \frac{4}{9}, 1\}$ has no sign change. So, there exists a stabilizing controller.

Example 2. An example is given below which illustrates the application of Theorem 3.1 as also the construction procedure to find the global stabilizing controller for the expanded system. The first open loop subsystem S_1 is described byS_1:

$$\begin{aligned} \dot{x}_1 &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_1 + \begin{bmatrix} 1 \\ 0 \end{bmatrix} v_1 \\ y_1 &= [1 \;\; 2]\, x_1 \\ w_1 &= [1 \;\; 1]\, x_1. \end{aligned} \tag{16}$$

As this is the only system to be stabilized, it is not required to satisfy any of the established conditions. Applying static output feedback $u_1 = -y_1$, we obtain the closed loop stable system $\bar{S}_1^c = S_1^c$:

$$\begin{aligned} \dot{x}_1 &= \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} x_1 + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ w_1 &= [1 \;\; 2]\, x_1. \end{aligned} \tag{17}$$

Now we connect the second subsystem given by S_2:

$$\begin{aligned} \dot{x}_2 &= \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} x_2 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_2 + \begin{bmatrix} 1 \\ 0 \end{bmatrix} v_2 \\ y_2 &= [-1 \;\; 1]\, x_2 \\ w_2 &= [-1 \;\; 1]\, x_2 \end{aligned} \tag{18}$$

with interconnection

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}. \tag{19}$$

The transfer functions $Z_{2,11}$, $Z_{2,12}$, $Z_{2,21}$ and $Z_{2,22}$ are computed as

$$Z_{2,11} = Z_{2,12} = Z_{2,21} = Z_{2,22} = \frac{s-1}{s^2+s+1}.$$

We factorize $Z_{2,22}$ as follows:

$$N_2 = \tilde{N}_2 = Z_{2,22}; \quad D_2 = \tilde{D}_2 = 1, \quad P_2 = \tilde{P}_2 = 0, \quad Q_2 = \tilde{Q}_2 = 1.$$

This implies that the class of all stabilizing controllers is charactrized as

$$u_2 = -R_2(1 - Z_{2,22}R_2)^{-1} y_2 \tag{20}$$

where $R_2 \in R_{ps}$. So we have

$$\begin{aligned} &T_{2,1} = T_{2,2} = T_{2,3} = \frac{s-1}{s^2+s+1} \\ &\psi_1 = \bar{E}_{2,1} T_1 \bar{E}_{1,2} = E_{21} T_1 E_{12} = \frac{1}{s+1} \\ &1 - \psi_1(T_{2,1} - T_{2,2} R_2 T_{2,3}) = 1 - \frac{1}{s+1}\left[\frac{s-1}{s^2+s+1} - \frac{(s-1)^2}{(s^2+s+1)^2} R_2\right]. \end{aligned} \tag{21}$$

Now, f and g contained in (21) are

$$f = 1 - \frac{1}{s+1}\frac{s-1}{s^2+s+1}; \quad g = \frac{1}{s+1}\frac{(s-1)^2}{(s^2+s+1)^2}.$$

It can be seen that $\sigma_1 = 1$, $\sigma_2 = \infty$, $m_1 = 2$ and $m_2 = 3$. As the sequence $\{f(\sigma_1),\ f(\sigma_2)\} = \{f(1),\ f(\infty)\} = \{\ 1, 1\ \}$ does not have any sign change, Theorem 3.1 guarantees the existence of a stabilizing controller.

Again, following the construction procedure given in [9, 10] with the unit interpolation algorithm , we get

$$\begin{aligned}(f+gR_2) = \Big\{&s^{10} + 10s^9 + 44s^8 + 113.3333s^7 + 192.469s^6 + 232.9877s^5 + 208.2469s^4\\ &+134.5679s^3 + 64.01234s^2 + 19.53086s + 3.851852\Big\}/\Big\{s^{10} + 10s^9 + 45s^8\\ &+120s^7 + 210s^6 + 252s^5 + 210s^4 + 120s^3 + 45s^2 + 10s + 1\Big\}\end{aligned}$$

which satisfies

$$\min_{R_2 \in R_{ps}} \delta(f+gR_2) = 0.$$

Substituting values of f and g in the above equation, the free parameter of the controller R_2 can be found. In fact, stabilizing controllers of lower orders do exist and cannot be obtained by existing interpolation algorithms.

Assume R_2 to be a real gain. Now,

$$\begin{aligned} f+gR_2 &= 1 - \frac{1}{s+1}\left[\frac{s-1}{s^2+s+1} - \frac{(s-1)^2}{(s^2+s+1)^2}R_2\right]\\ &= \frac{(s^5+3s^4+4s^3+5s^2+3s+2)+(s-1)^2R_2}{(s+1)(s^2+s+1)^2}. \end{aligned} \tag{22}$$

Root locus method can be used to investigate the stability of the numerator of the transfer function above. This would give the range of R_2 for $(f+gR_2)$ to be a unit, if any. It is seen that

$$0 < R_2 < 0.2720778$$

satisfies the requirement. Similarly, assuming $R_2 = \frac{K}{s+1}$ we have,

$$\begin{aligned} f+gR_2 &= 1 - \frac{1}{s+1}\left[\frac{s-1}{s^2+s+1} - \frac{(s-1)^2}{(s^2+s+1)^2}\frac{K}{(s+1)}\right]\\ &= \frac{(s+1)(s^2+s+1)[(s+1)(s^2+s+1)-(s-1)]+(s-1)^2K}{(s+1)^2(s^2+s+1)^2}. \end{aligned} \tag{23}$$

From the root loci of the fictitious plant in the numerator of this transfer function, it is seen that for $0 < K < 0.75$, $f + g\frac{K}{s+1}$ is R_{ps}-unimodular and consequently a stabilizing controller exists with the free parameter of order 1.

4. MULTIVARIABLE CASE

Before proving the main result for the general MIMO case, which is not a very straightforward extension of the scalar case, a few Lemmas are first presented. The proof of the main Theorem utilizes all these results. These Lemmas are belived to be of independent interest and can be used to solve other problems as well.

Lemma 4.1. Consider the following matrices $C^{n\times p}$, $R^{p\times m}$, $B^{m\times n}$, $A^{n\times n} \in M(R_{ps})$. The following identitiy holds.

$$a^p\ |aI_n - CRBA^{adj}| = a^n\ |aI_p - RBA^{adj}C|$$

where $a = |A|$.

Proof. Consider the matrix multiplication

$$\begin{bmatrix} I_n & -C \\ 0 & I_p \end{bmatrix} \begin{bmatrix} aI_n & C \\ RBA^{adj} & I_p \end{bmatrix} = \begin{bmatrix} aI_n - CRBA^{adj} & 0 \\ RBA^{adj} & I_p \end{bmatrix}.$$

Taking determinants on both sides we have

$$\begin{vmatrix} aI_p & C \\ RBA^{adj} & I_p \end{vmatrix} = |aI_n - CRBA^{adj}|.$$

Again consider,

$$\begin{bmatrix} I_n & 0 \\ -RBA^{adj} & aI_p \end{bmatrix} \begin{bmatrix} aI_n & C \\ RBA^{adj} & I_p \end{bmatrix} = \begin{bmatrix} aI_n & C \\ 0 & aI_p - RBA^{adj}C \end{bmatrix}.$$

Taking determinants on both sides

$$a^p|aI_n - CRBA^{adj}| = a^n|aI_p - RBA^{adj}C|. \qquad \square$$

Lemma 4.2. Consider the matrices $A^{n\times n}$, $C^{n\times p}$, $R^{p\times m}$ and $B^{m\times n} \in M(R_{ps})$. If (A, C) is left-coprime and (B, A) is right-coprime then there exist appropriate square matrices $A_1^{n\times n}, C_1^{n\times n}, R_1^{n\times n}$ and $B_1^{n\times n} \in M(R_{ps})$ such that $|A + CRB| = |A_1 + C_1R_1B_1|$, (A_1, C_1) is left-coprime and (B_1, A_1) is right-coprime.

Proof. The proof is divided into two parts. First, the cases of $p > n$ and $p < n$ are shown to be equivalent to $p = n$.

- Suppose $p > n$, then there exist unimodular matrices U and V in R_{ps} such that

$$UCV = \begin{array}{cc} \scriptstyle n & \scriptstyle p-n \\ \left[C' \right. & \left. 0 \right] \end{array} n.$$

As (A, C) is left-coprime

$$\begin{aligned} AX + CY &= I \\ UAX + UCVV^{-1}Y &= U \\ UAU^{-1}UX + \begin{array}{cc} \scriptstyle n & \scriptstyle p-n \\ \left[C' \right. & \left. 0 \right] \end{array} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} &= U \\ A'X_1 + C'Y_1 &= U \end{aligned}$$

where $A' := UAU^{-1}$, $X_1 := UX$ and $V^{-1}Y := \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}$.

So, (A', C') is left-coprime. Hence,

$$\begin{aligned} |A + CRB| &= |UAU^{-1} + UCVV^{-1}RBU^{-1}| \\ &= |A' + [C' \;\; 0]\begin{bmatrix} R' \\ R'' \end{bmatrix} B'| \\ &= |A' + C'R'B'| \end{aligned}$$

where $V^{-1}R$ is partitioned as $\begin{bmatrix} R' \\ R'' \end{bmatrix}$ and $B' := BU^{-1}$.

- Suppose $p < n$. As (A,C) is left coprime

$$\begin{aligned} AX + CY &= I \\ AX + [\overset{p}{C} \;\; \overset{n-p}{0}]\begin{bmatrix} Y \\ 0 \end{bmatrix} &= I \\ A'X + C'Y' &= I \end{aligned} \tag{24}$$

where $C' := \begin{bmatrix} \overset{p}{C} & \overset{n-p}{0} \end{bmatrix}$, $Y' := \begin{bmatrix} Y \\ 0 \end{bmatrix}$ and $A' := A$. So, (A', C') is left-coprime.

$$\begin{aligned} |A + CRB| &= \left| A' + [C \;\; 0]\begin{bmatrix} R \\ 0 \end{bmatrix} B \right| \\ &= |A' + C'R'B'| \end{aligned}$$

where $C' := [C \;\; 0]$, $R' := \begin{bmatrix} R \\ 0 \end{bmatrix}$ and $B' := B$.

Now, it is shown that the cases $m > n$ and $m < n$ are equivalent to $m = n$.

- Suppose $m > n$, then there exist unimodular matrices U and V in R_{ps} such that

$$UB'V = \begin{bmatrix} B_1 \\ 0 \end{bmatrix} \begin{matrix} n \\ n - m \end{matrix}.$$

As (B', A') is right-coprime

$$\begin{aligned} XA' + YB' &= I \\ XA'V + YB'V &= V \\ XA'V + YU^{-1}UB'V &= V \\ XA'V + YU^{-1}\begin{bmatrix} B_1 \\ 0 \end{bmatrix} &= V \\ XA'V + [Y_1 \;\; Y_2]\begin{bmatrix} B_1 \\ 0 \end{bmatrix} &= V \\ XVV^{-1}A'V + Y_1B_1 &= V \\ X_1A_1 + Y_1B_1 &= V \end{aligned}$$

where $A_1 := V^{-1}A'V$, $X_1 := XV$ and YU^{-1} is partitioned as $[Y_1 \;\; Y_2]$. Now,

$$\begin{aligned} |A+CRB| &= |A'+C'R'B'| \\ &= |V^{-1}A'V+V^{-1}C'R'B'V| \\ &= |A_1+C_1R'U^{-1}UB'V| \\ &= \left|A_1+C_1[R_1 \;\; R_2]\begin{bmatrix} B_1 \\ 0 \end{bmatrix}\right| \\ &= |A_1+C_1R_1B_1| \end{aligned}$$

where $C_1 := V^{-1}C'$ and $R'U^{-1}$ is partitioned as $[R_1 \;\; R_2]$.

– Suppose $m < n$. As (B', A') is right-coprime,

$$\begin{aligned} XA'+YB' &= I \\ XA'+[Y \;\; 0]\begin{bmatrix} B' \\ 0 \end{bmatrix} &= I \\ XA'+Y_1B_1 &= I \end{aligned}$$

where $Y_1 := [Y \;\; 0]$ and $B_1 := \begin{bmatrix} B' \\ 0 \end{bmatrix}$. Now,

$$\begin{aligned} |A+CRB| &= |A'+C'R'B'| \\ &= \left|A'+C'[R' \;\; 0]\begin{bmatrix} B' \\ 0 \end{bmatrix}\right| \\ &= |A'+C'R_1B_1| \\ &= |A_1+C_1R_1B_1| \end{aligned}$$

where $A_1 := A'$, $C_1 := C'$, $R_1 := [R' \;\; 0]$ and $B_1 := \begin{bmatrix} B' \\ 0 \end{bmatrix}$. □

Remark. Henceforth, only the case (C, A, B) all square is considered in all the results established below. This is due to the fact that the cases of $m \neq n$ and $p \neq n$ have been shown to be equivalent to $m = p = n$.

Lemma 4.3. Suppose $A^{n\times n}$, $C^{n\times n}$ and $B^{n\times n} \in M(R_{ps})$ are given such that (B, A) is right coprime and (A, C) is left coprime. In other words, (B, A, C) is a bicoprime factorization. Then there exists an $R \in R_{ps}$ such that $|A+CRB| \neq 0$.

Proof. If $|A| \neq 0$, let $R = 0$. Otherwise, proceed as follows. For any arbitrary unimodular matrices U and $V \in M(R_{ps})$ we have,

$$\begin{aligned} |A+CRB| &= |U||A+CRB||U^{-1}| \\ &= |UAU^{-1}+UCRBU^{-1}| \\ &= |UAU^{-1}+UCVV^{-1}RBU^{-1}| \\ &= |A_1+C_1R_1B_1| \end{aligned}$$

where $A_1 := UAU^{-1}, C_1 := UCV, R_1 := V^{-1}R$ and $B_1 := BU^{-1}$. Since C and A are left-coprime, C_1 and A_1 are also left-coprime. In other words, for X and $Y \in M(R_{ps})$,

$$\begin{aligned} AX + CY &= I \\ &\Rightarrow UAU^{-1}UX + UCVV^{-1}Y = U \\ &\Rightarrow A_1X_1 + C_1Y_1 = U \end{aligned}$$

with $X_1 := UX, Y_1 := V^{-1}Y$ and U, a unimodular matrix in $M(R_{ps})$, i.e., $\exists\, X_2, Y_2 \in M(R_{ps})$ that satisfy the Bezout identity: $A_1X_2 + C_1Y_2 = I$ where $X_2 = X_1U^{-1}$ and $Y_2 = Y_1U^{-1}$. Using similar arguments, it can be shown that A_1 and B_1 are right-coprime if A and B are so. The unimodular matrices U and $V \in M(R_{ps})$ can now be chosen in such a way that $C \in R_{ps}^{n\times n}$ is converted to its Smith form C_1:

$$C_1 := UCV \tag{25}$$

where $C_1 := \mathrm{Diag}(c_1, c_2, \ldots, c_k, c_{k+1}, \ldots, c_n)$ with elements c_{k+1} to c_n all zeros. Note that k is the rank of C alone. Since C_1 and A_1 are left-coprime, the matrix $[A_1 \quad C_1] \in R_{ps}^{n\times 2n}(s)$ has full row rank which implies that the $k+1$st to nth rows of A_1 are independent. Suppose that among the rows 1 to k of A_1, i_1 to i_r rows are dependent. Again, due to right-coprimeness of A_1 and B_1, the matrix $\begin{bmatrix} A_1 \\ B_1 \end{bmatrix}$ has full rank. As i_1 to i_r rows of A_1 are dependent, the minor of order n obtained by excluding i_1 to i_r rows of A_1 and including j_1 to j_r rows of B_1 is nonzero. Then, using Binet-Cauchy formula it can be shown that $|A_1 + C_1R_1B_1| \neq 0$ for R_1 with elements $r_{i_x,j_x} = 1$ where $x = 1$ to r and $r_{i,j} = 0$ for all other i's and j's. So, $|A_1 + C_1R_1B_1| = c_{i_1}, \ldots, c_{i_r}t$ where t is the determinant with all but $i_1, \ldots, i_r$ numbered rows of A and $j_1, \ldots, j_r$ numbered rows of B. So, $|A_1 + C_1R_1B_1| = |A + CRB|$, where $R = VR_1$. □

Lemma 4.4. Suppose $P \in M(R(s))$ and (B, A, C) is a bicoprime factorization, i.e., (i) $|A| \neq 0$ and $P = BA^{-1}C$, (ii) (B, A) is a right coprime factorization and (iii) (A, C) is left coprime factorization. Consider any left and right coprime factorizations $(\tilde{A}_1, \tilde{B})$ and $(\tilde{C}, \tilde{A}_2)$ of (B, A) and (A, C) respectively. Then $(B\tilde{C}, \tilde{A}_2)$ is right coprime and $(\tilde{A}_1, \tilde{B}C)$ is left coprime.

Proof.
$$P = BA^{-1}C = \tilde{A}_1^{-1}\tilde{B}C.$$

Since (B, A) is right coprime and $(\tilde{A}_1, \tilde{B})$ is left coprime $\exists\, X, Y, \tilde{X}, \tilde{Y} \in M(R_{ps})$ such that

$$\begin{bmatrix} \tilde{A}_1 & \tilde{B} \\ \tilde{Y} & -\tilde{X} \end{bmatrix} \begin{bmatrix} X & -B \\ Y & A \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \tag{26}$$

or,

$$\tilde{A}_1X + \tilde{B}Y = I \tag{27}$$
$$\tilde{A}_1B = \tilde{B}A \tag{28}$$
$$\tilde{Y}B + \tilde{X}A = -I \tag{29}$$
$$\tilde{Y}X = \tilde{X}Y. \tag{30}$$

Since (A, C) is left coprime, $\exists X_1, Y_1 \in M(R_{ps})$

$$AX_1 + CY_1 = I. \tag{31}$$

Now the problem is to find $X_2, Y_2 \in M(R_{ps})$ so that

$$\tilde{B}CX_2 + \tilde{A}_1 Y_2 = I$$

i. e., the pair $(\tilde{A}_1, \tilde{B}C)$ is left coprime.
Define

$$\begin{aligned} X_2 &:= -Y_1 Y & (32)\\ Y_2 &:= -X - BX_1 Y. & (33)\end{aligned}$$

So,

$$\begin{aligned} \tilde{B}CX_2 + \tilde{A}_1 Y_2 &= -\tilde{B}CY_1 Y - \tilde{A}_1 X - \tilde{A}_1 BX_1 Y \\ &= -\tilde{B}Y + \tilde{B}AX_1 Y - \tilde{A}_1 X - \tilde{A}_1 BX_1 Y \\ &= -\tilde{B}Y - \tilde{A}_1 X + (\tilde{B}A - \tilde{A}_1 B)X_1 Y \\ &= -I & (34)\end{aligned}$$

which implies that $(\tilde{A}_1, \tilde{B}C)$ is left coprime. □

Theorem 4.1. Suppose $(B, A, C) \in R_{ps}^{n\times n}$ is a bicoprime factorization of $P = BA^{-1}C \in M(R(s))$. In other words, (i) $|A| \neq 0$ and $P = BA^{-1}C$, (ii) (B, A) is a right coprime factorization and (iii) (A, C) is left coprime factorization. Consider any left and right coprime factorizations $(\tilde{A}_1, \tilde{B})$ and $(\tilde{C}, \tilde{A}_2)$ of (B, A) and (A, C) respectively. Let 'a'$= |A|$; $\tilde{z_1}$ and z_1 be the smallest invariant factors of $\tilde{B}C$ and $B\tilde{C}$ respectively. Then, the sets

- $\min_{R\in M(R_{ps})} \delta(|A + CRB|)$,
- $\min_{R\in M(R_{ps})} \delta(|\tilde{A}_2 + RB\tilde{C}|)$,
- $\min_{R\in M(R_{ps})} \delta(|\tilde{A}_1 + \tilde{B}CR|)$,
- $\min_{r\in R_{ps}} \delta(a + rz_1) =: I(a, z_1)$,
- $\min_{r\in R_{ps}} \delta(a + \tilde{z_1} r) =: I(a, \tilde{z_1})$

are equal where $I(a, \tilde{z_1})$ denotes the number of sign changes in the sequence $\{a(\sigma_1), a(\sigma_2), \ldots, a(\sigma_n)\}$; $\{\sigma_1, \sigma_2, \ldots, \sigma_n\}$ being the distinct nonnegative real zeros of 'z_1' or '$\tilde{z_1}$'in ascending order including ∞.

Proof. From Theorem 4.4.1 in [9], statements 2 and 4 are known to be equivalent. Similarly, statements 3 and 5 are also equivalent. Only the equivalence of statements

1, 2 and 3 is left to be proved. The statements 1 and 3 are shown to be equivalent below.

$$\begin{aligned}|A+CRB| &= |A||I+A^{-1}CRB| \\ &= |A||I+BA^{-1}CR| \\ &= |A||I+\tilde{A_1}^{-1}\tilde{B}CR| \\ &= |A||\tilde{A_1}^{-1}||\tilde{A}_1+\tilde{B}CR| \\ &= |\tilde{A}_1+\tilde{B}CR|.\end{aligned}$$

Since $|A| \sim |\tilde{A}_1$, without loss of generality, the equivalence is assumed to be an equality [9]. Following the same procedure, statements 1 and 2 can be shown to be equivalent implying equivalence of statements 2 and 3 too. □

Algorithm for designing the free parameter R in the multivariable case

- Find z_1, $\tilde{z_1}$ (smallest invariant factors) and a (characteristic determinant) from $B\tilde{C}$, $\tilde{B}C$ and A respectively, where B, C, $\tilde{B}$, $\tilde{C}$ and A are as defined in Theorem 4.1.
- Find an r such that $a+rz_1$ or $a+\tilde{z_1}r$ is unimodular in R_{ps} using the interpolation algorithm given in [9, 10].
- Convert $\tilde{A_1}^{adj}\tilde{B}C$ or $B\tilde{C}\tilde{A_2}^{adj}$ to its Smith form, i. e.,

$$X := U\tilde{A_1}^{adj}\tilde{B}CV = \text{Diag}(x_1, x_2, \ldots, x_k, 0, \ldots, 0).$$

- Find $q_1, q_2, \ldots, q_k \in R_{ps}$ such that

$$q_1x_1a^{k-1} + q_2x_1x_2a^{k-2} + \cdots + q_kx_1\cdots x_k = a^{k-1}\tilde{z_1}r$$

- Define the bordered companion matrix

$$R_0 := \begin{bmatrix} q_1 & q_2 & \cdots & q_k & 0 & \cdots & 0 \\ -1 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & -1 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \end{bmatrix} \in R_{ps}^{p\times m}(s).$$

- $R = V^{-1}R_0$.

Lemma 4.5. Suppose A, B and $C \in M(R_{ps})$ are all square with $|B| \neq 0$ and $|C| \neq 0$. Then $\exists\, R \in M(R_{ps})$ such that $\delta(|A+CRB|) < \delta(|BC|)$.

Proof. Case (i): If $|A| = 0$, then the above inequality is satisfied with R=0.
Case (ii): Suppose $|A| \neq 0$.

Let F_1 be a greatest common right divisor of A and B, i.e., $A = A_1F_1$ and $B = B_1F_1$. Also, let F_2 be a gretaest common left divisor of A_1 and C, i.e., $A_1 = F_2A_2$ and $C = F_2C_1$. Let $a_2 = |A_2|$ and $z_1, \tilde{z_1}$ be smallest invariant factors of matrices B_1C_{1r} and $B_{1l}C_1$ respectively where (A_{2r}, C_{2r}) and (B_{1l}, A_{2l}) are left and right coprime factorizations of the pairs (C_1, A_2) and (A_2, B_1). From the basic principles of a Proper Euclidean Domain it follows that for some $R \in M(R_{ps})$,

$$\begin{aligned}
\delta(|A+CRB|) &= \delta(|F_2|) + \delta(|A_2 + C_1RB_1|) + \delta(|F_1|) \\
&= \delta(|F_2|) + \delta(|A_{2r} + RB_1C_{1r}|) + \delta(|F_1|) \\
&= \delta(|F_2|) + \delta(|A_{2l} + B_{1l}C_1R|) + \delta(|F_1|) \\
&= \delta(|F_2|) + I(a_2, z_1) + \delta(|F_1|) \quad \text{(Theorem 4.1)} \\
&\le \delta(|F_2|) + \delta(z_1) + \delta(|F_1|) \\
&\le \delta(|F_2|) + \delta(|B_1C_{1r}|) + \delta(|F_1|) \\
&= \delta(|F_2|) + \delta(|C_{1r}B_1|) + \delta(|F_1|) \\
&= \delta(|F_2|) + \delta(|C_{1r}B|) \\
&= \delta(|F_2|) + \delta(|C_1B|) \\
&= \delta(|CB|) \\
&= \delta(|BC|).
\end{aligned} \tag{35}$$

□

Theorem 4.2. Consider the fictitious plant $P = \bar{\psi}_{N-1}T_{N2}(I - \bar{\psi}_{N-1}T_{N-1})^{-1}T_{N3}$ and define $\hat{B} := \bar{\psi}_{N-1}T_{N2}; \hat{A} := (I - \bar{\psi}_{N-1}T_{N1}); \hat{C} := T_{N3}$. Let F_1 be the greatest common right divisor of $(\hat{B}, \hat{A})$, i.e., $\hat{A} = \hat{A}_0F_1$ and $\hat{B} = BF_1$. Let F_2 be greatest common left divisor of $(\hat{A}_0, \hat{C})$, i.e., $\hat{A}_0 = F_2A$ and $\hat{C} = F_2C$. So, $(B, A, C, 0)$ is a bicoprime factorization [9]. Now consider a left and a right coprime factorization $(\tilde{A_1}, \tilde{BC})$ and $(\tilde{BC}, \tilde{A_2})$ respectively of $(B, A, C, 0)$. Let $\sigma_1, \sigma_2, \ldots, \sigma_l$ be the real nonnegative distinct blocking zeros including ∞ of $\tilde{BC}$ (or, $\tilde{BC}$) arranged in the ascending order. Then,

-

$$\min_{R_N \in M(R_{ps})} \delta(|I - \bar{\psi}_{N-1}T_{N1} + \bar{\psi}_{N-1}T_{N2}R_NT_{N3}|) = \delta(|F_1|) + \delta(|F_2|) \tag{36}$$

- The following real numbers have the same sign.

$$\{f(\sigma_1), f(\sigma_2), \ldots, f(\sigma_l)\}$$

where $f := |\tilde{A_1}|$ or $|\tilde{A_2}|$. Moreover, the stabilizing controller for the Nth subsystem which also stabilizes the expanded system $\tilde{S}_N^c$ exists if and only if (i) $\delta|F_1| = \delta(|F_2|) = 0$ and (ii) the above sequence $\{f(\sigma_1), f(\sigma_2), \ldots, f(\sigma_l)\}$ has the same sign.

The existence of a stabilizing controller for the multivariable plant can be checked using the above Theorem.

5. SUFFICIENT CONDITION FOR A BROADER CLASS OF SYSTEMS

The class of subsystems treated in [8] does not allow direct transmissions from interconnection inputs (v_i's) to the interconnection outputs (w_i's). In fact, according to them, "In this case, the state of the overall system may not be the collection of subsystem states and we will have critical problems in analysing the system behaviour". We would like to point out that the larger class of subsystems mentioned above implies that if $Z_{N11} = [A_N, G_N, H_N, F_{4N}]$, then $T_{N1} = Z_{N11} - Z_{N12}\tilde{P}_N\tilde{D}_N Z_{N21}$ is generically biproper; the nongeneric case being only when $Z_{N12}\tilde{P}_N\tilde{D}_N Z_{N21}(\infty)$ is equal to F_{4N}.

So Theorem 3.1 of [8] is modified to handle this extended class of systems. We present the modified version of the theorem below.

Theorem 4.3. There exists a local controller LC_N for which both the closed loop subsystem S_N^c and the expanded system $\bar{S}_N^c$ are stable if the equations

$$T_{N2}X_N = T_{N1}$$

and

$$Y_N T_{N3} = T_{N1}$$

have solutions in $M(R_{ps})$, (i.e., not just in R_s).

Proof. Case (i) Solution in $M(R_{ps})$:
Suppose that the solution exists in $M(R_{ps})$. If the equations

$$\begin{aligned} T_{N2}X_N &= T_{N1} \\ Y_N T_{N3} &= T_{N1} \end{aligned} \tag{37}$$

have solutions in $M(R_{ps})$, then

$$(I-\bar{\psi}_{N-1}T_{N1}+\bar{\psi}_{N-1}T_{N2}R_N T_{N3}) = (I-\bar{\psi}_{N-1}T_{N1}+\bar{\psi}_{N-1}T_{N2}X_N R'_N Y_N T_{N3}) \tag{38}$$

where

$$R_N := X_N R'_N Y_N \in M(R_{ps})$$

if R'_N is in $M(R_{ps})$. From (37), (38) is

$$(I - \bar{\psi}_{N-1}T_{N1} + \bar{\psi}_{N-1}T_{N1}R'_N T_{N1}). \tag{39}$$

Now from Lemma 4.1

$$\begin{aligned} & (|I - \bar{\psi}_{N-1}T_{N1} + \bar{\psi}_{N-1}T_{N1}R'_N T_{N1}|) \\ = & (|I + \bar{\psi}_{N-1}T_{N1}R'_N T_{N1}(I - \bar{\psi}_{N-1}T_{N1})^{-1}||I - \bar{\psi}_{N-1}T_{N1}|) \\ = & (|I + R'_N T_{N1}(I - \bar{\psi}_{N-1}T_{N1})^{-1}\bar{\psi}_{N-1}T_{N1}||I - \bar{\psi}_{N-1}T_{N1}|) \\ = & (|I + R'_N T_{N1}\bar{\psi}_{N-1}T_{N1}(I - \bar{\psi}_{N-1}T_{N1})^{-1}||I - \bar{\psi}_{N-1}T_{N1}|) \\ = & (|I - \bar{\psi}_{N-1}T_{N1} + R'_N T_{N1}\bar{\psi}_{N-1}T_{N1}|). \end{aligned}$$

Now from the proof of Lemma 5.5.6 in [9], it follows that there always exists an R'_N such that (39) and hence (38) is R_{ps}-unimodular.

Case (ii) Solution not just in $M(R_s)$.

This part is proved by giving a counter-example. It is not sufficient for the solution to belong to $M(R_s)$. It must belong to $M(R_{ps})$. Let us consider the scalar case.

$$\bar{\psi}_{N-1} = 1; \; T_{N1} = \frac{(s-1)(2s-1)}{(s+1)^2}; \; T_{N2} = \frac{(s-1)}{(s+1)^2}; \; T_{N3} = \frac{(2s-1)}{(s+1)^2}.$$

Solving for X_N and Y_N, we get

$$X_N = \frac{(2s-1)}{1}; \; Y_N = \frac{(s-1)}{1}.$$

It is clear that both X_N and Y_N are in $M(R_s)$. However, at the simple real zeros of $\bar{\psi}_{N-1}T_{N2}T_{N3}$, i. e., at $\{0.5, 1, \infty\}$, $(I - \bar{\psi}_{N-1}T_{N1})$ has the values $(1, 1, -1)$, showing a change in sign in the sequence. Thus the condition given in Theorem 3.1 implies δ to be 1 because of only one sign change. This in turn implies that for any local controller, the expanded system will have at least one unstable closed loop pole. □

Remark. The condition derived in [8] requires checking for solution of (37) in $M(R_s)$ only because the class of systems considered there does not have direct feedthrough terms.

6. CONCLUSIONS

The criterion established in this paper for the Expanding System Problem is both necessary and sufficient. Moreover, it is much simpler from the computational point of view than that of Tan and Ikeda [8]. The aforesaid comments hold true for the sufficient condition developed in Section 4 for a larger class of systems involving direct feedthrough terms. The proposed necessary and sufficient condition is also applicable to the same class. In fact, the theory developed here also solves the problem of strong stabilization of a plant $P = BA^{-1}C$, where (B, A, C) is a bicoprime factorization. This important problem has applications such as *reliable decentralized control.*

The order of the controller for the ESP using the proposed algorithm is very high and in fact, much lower order controllers might exist as detailed in the illustrative example. However, it is not known what systematic procedure will lead to the design of the least order controller. This may constitute a research problem of future investigation. Some of the results in this paper are generalizations of algebraic design techniques for reliable stabilization established in [9].

NOTATIONS

- $R_s(s), R_{ps}$ Ring of proper stable rational functions.
- $R_s(s), R_s$ Ring of stable rational functions.
- $M(X)$ Matrices over the ring X, where X can be either R_{ps} or R_s.

- $X^{p \times q}$ Matrices of dimension $p \times q$ over the ring X.
- $deg(x)$ Degree of a polynomial x.
- $\delta(t)$ Degree of $t \in M(R_{ps})$ = number of right half plane zeros of t including ∞. This degree function makes $M(R_{ps})$ a Proper Euclidean Domain.
- $|A|$ Characteristic Determinant of $A \in M(R_{ps})$
- $\sim$ Equivalence relation.

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