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CONSTRUCTIONS OF LOWER AND UPPER SOLUTIONS FOR A NONLINEAR BOUNDARY VALUE PROBLEM OF THE THIRD ORDER AND THEIR APPLICATIONS

JÁN RUSNÁK

1. Introduction

In this paper we shall study the following boundary value problem (BVP, for short):

$$x''' = f(t, x, x', x''), \ (t, x, x', x'') \in I \times R^3, \ I = [a_1, a_3], \tag{1}$$
$$f \text{ is continuous on } I \times R^3,$$

$$\begin{aligned} \alpha_2 x'(a_1) - \alpha_3 x''(a_1) &= A_1 \\ \beta_1 x(a_2) + \beta_2 x'(a_2) - \beta_3 x''(a_2) &= A_2 \\ \gamma_2 x'(a_3) + \gamma_3 x''(a_3) &= A_3, \end{aligned} \tag{2}$$

$$\alpha_i, \beta_i, \gamma_i \geqq 0, \ i = 2, 3, \ \beta_1 > 0, \ \alpha_2 + \alpha_3 > 0,$$
$$\gamma_2 + \gamma_3 > 0, \ \alpha_2 + \gamma_2 > 0, \ a_1 < a_2 < a_3.$$

Denote $h = a_3 - a_1$, $h_1 = a_2 - a_1$, $h_2 = a_3 - a_2$.

The associated homogeneous BVP, i.e. the problem for the equation

$$x''' = 0 \tag{3}$$

with boundary conditions obtained from (2) for $A_1 = A_2 = A_3 = 0$, has only the trivial solution because

$$\begin{aligned} \Delta = \begin{vmatrix} 0, & \alpha_2, & 2\alpha_2 a_1 - 2\alpha_3 \\ \beta_1, & \beta_1 a_2 + \beta_2, & \beta_1 a_2^2 + 2\beta_2 a_2 - 2\beta_3 \\ 0, & \gamma_2, & 2\gamma_2 a_3 + 2\gamma_3 \end{vmatrix} = \\ = -2\beta_1(\alpha_2\gamma_2 h + \alpha_2\gamma_3 + \alpha_3\gamma_2) < 0. \end{aligned} \tag{4}$$

Let $G_1(t, s)$ and $G_2(t, s)$ be Green's functions in the sense of [1] and [2], corresponding to the BVP considered here. The functions G_1, G_2 expressed explicitly can be found in [7].

Further, if $\varphi(t)$ is a solution of the BVP (3) and (2), then the solution $x(t)$ of the BVP (1) and (2) is a solution of the integro-differential equation

$$x(t) = \varphi(t) + \sum_{k=1}^{2} \int_{a_k}^{a_{k+1}} G_k(t, s) f(s, x(s), x'(s), x''(s)) \, \mathrm{d}s, \tag{5}$$

and vice versa.

The method based on suitably defined lower and upper solutions is used when investigating the solutions of nonlinear bundary value problems. Lower and upper solutions play significant roles, e.g., when searching for an approximate solution. Namely, one can construct successive approximations converging to the solution one is searching for, employing a concrete lower and upper solution as starting data (cf. e.g. [3]—[6], [10] and [11]).

In this paper, we shall show how to construct certain lower and upper solutions of the BVP (1) and (2) for large class of functions f, and how to apply these to get some existence theorems.

2. An existence theorem via lower and upper solutions

Denote the boundary conditions (2) formally by $B\{x, =\}$.

A function $\alpha \in C_3(I)$ will be called a lower solution of the BVP (1) and (2) if

$$\alpha''' \geqq f(t, \alpha, \alpha', \alpha''), \qquad B\{\alpha, \leqq\}. \tag{6}$$

Similarly, $\beta \in C_3(I)$ is an upper solution, provided

$$\beta''' \leqq f(t, \beta, \beta', \beta''), \qquad B\{\beta, \geqq\}. \tag{7}$$

Let for α, β

$$\alpha(a_1) \leqq \beta(a_1), \ \alpha'(t) \leqq \beta'(t) \text{ hold on } I. \tag{8}$$

Put

$$\delta(y_1, y_2, y_3) = \begin{cases} y_1, & y_2 < y_1 \leqq y_3 \\ y_2, & y_1 \leqq y_2 \leqq y_3, \\ y_3 & y_1 \leqq y_3 < y_2 \end{cases} \quad y_1, y_2, y_3 \in R.$$

Let the function f be modified on $I \times R^3$ to get the following F:

$$F(t, x, x', x'') = f(t, \delta(\alpha(t), x, \beta(t)), \delta(\alpha'(t), x', \beta'(t)), x'') + \\ + \frac{x' - \delta(\alpha'(t), x', \beta'(t))}{1 + x'^2}.$$

When f is bounded on $I \times R^3$, then

$$M_1 + M_3 \leqq F(t, x, x', x'') \leqq M_2 + M_4 \text{ on } I \times R^3,$$

where

$$M_1 = \inf_{I \times R^3} f(t, x, x', x''), \quad M_2 = \sup_{I \times R^3} f(t, x, x', x''),$$

$$M_3 = \min_{\substack{t \in I \\ x' < \alpha'(t)}} \frac{x' - \alpha'(t)}{1 + x'^2} < 0, \quad 0 < M_4 = \max_{\substack{t \in I \\ x' > \beta'(t)}} \frac{x' - \beta'(t)}{1 + x'^2}. \tag{9}$$

Put $m_\alpha = \max_I \alpha'(t)$, $m_\beta = \min_I \beta'(t)$; then

$$M_3 = -\frac{1}{2}\left(\sqrt{m_\alpha^2 + 1} + m_\alpha\right), \quad M_4 = \frac{1}{2}\left(\sqrt{m_\beta^2 + 1} - m_\beta\right).$$

Now consider the differential equation

$$x''' = F(t, x, x', x''). \tag{10}$$

According to [7, Thm. 1] there exists, if f is bounded, at least one solution of the BVP (10) and (2). In the following lemma we find a lower and an upper estimate of the initial value $x(a_1)$ of the solution x to this BVP.

From (5), we get for $x(a_1)$:

$$x(a_1) = \varphi(a_1) + \sum_{k=1}^{2} \int_{a_k}^{a_{k+1}} G_k(a_1, s) F(s, x(s), x'(s), x''(s))\, \mathrm{d}s,$$

$$G_1(a_1, s) = -\frac{1}{\Delta}(\Delta_{11} + \Delta_{12} a_1 + \Delta_{13} a_1^2)(\alpha_2(s - a_1) + \alpha_3) - \frac{1}{2}(s - a_1)^2,$$

$$s \in (a_1, a_2),$$

$$G_2(a_1, s) = -\frac{1}{\Delta}(\Delta_{11} + \Delta_{12} a_1 + \Delta_{13} a_1^2)(\alpha_2(s - a_1) + \alpha_3) +$$

$$+ \frac{1}{2}(s - a_2)^2 - \frac{\beta_2}{\beta_1}(s - a_2) - \frac{\beta_3}{\beta_1} - \frac{1}{2}(s - a_1)^2, \quad s \in (a_2, a_3)$$

where Δ_{ij}'s are the minors of the determinant Δ from (4) (with appropriate signs). The following holds:

$$\Delta_{11} + \Delta_{12} a_1 + \Delta_{13} a_1^2 = \beta_1 \gamma_2 (h^2 - h_2^2) + 2\beta_1 \gamma_3 h + 2\beta_2 \gamma_2 h_2 + 2\beta_2 \gamma_3 + 2\beta_3 \gamma_2 \geqq 0.$$

The results above imply:

Lemma 1. *Let f be bounded on $I \times R^3$ and let x be an arbitrary solution of the BVP* (10) *and* (2). *Then*:

$$k_1 \leqq x(a_1) \leqq k_2, \tag{11}$$

where

$$k_1 = \varphi(a_1) - (M_1 + M_3)\frac{h}{\Delta}(\Delta_{11} + \Delta_{12}a_1 + \Delta_{13}a_1^2)\left(\frac{\alpha_2}{2}h + \alpha_3\right) +$$

$$+ (M_2 + M_4)\left(\frac{1}{6}h_2^3 - \frac{\beta_2}{2\beta_1}h_2^2 - \frac{\beta_3}{\beta_1}h_2 - \frac{1}{6}h^3\right) \tag{12}$$

and k_2 is of the same form as k_1, but the parentheses $(M_1 + M_3)$ and $(M_2 + M_4)$ are replaced by each other; the constants M_1, M_2, M_3, M_4 are those from (9).

Lemma 2. (Lemma 6 in [7]). *Let the functions $\alpha, \beta \in C_3(I)$ satisfy* (8) *(α and β need not be a lower and an upper solution, respectively) and let there exist a positive constant L such that*

$$|f(t, x, x', x'') - f(t, x, x', y'')| \leqq L|x'' - y''|,$$

$$(t, x, x') \in \omega = \{(t, x, x': t \in I, \alpha(t) \leqq x \leqq \beta(t), \tag{13}$$

$$\alpha'(t) \leqq x' \leqq \beta'(t)\}, x'', y'' \in R.$$

Then there exists a positive constant R_2 such that for any solution $x(t) \in C_3(I)$ of the equation (1) *which satisfies the conditions*

$$\alpha(t) \leqq x(t) \leqq \beta(t), \ \alpha'(t) \leqq x'(t) \leqq \beta'(t) \text{ on } I, \tag{14}$$

$$|x''(t)| \leqq R_2 \text{ holds on } I.$$

This lemma is still true when $\alpha, \beta \in C_2(I)$.

In [7] it has been derived how to compute and estimate the constant R_2. This R_2 satisfies the equation

$$\int_{2R_1/h}^{R_2} \frac{s \, ds}{Ls + m} = 2R_1, \tag{15}$$

where

$$R_1 \leqq \max(\max_I |\alpha'(t)|, \max_I |\beta'(t)|),$$

$$m \leqq \max_\omega (L|\beta''(t)| + |f(t, x, x', \beta''(t))|).$$

The following estimate of R_2 can be verified:

$$R_2 < 2LR_1 + m + 2R_1/h. \tag{16}$$

Using Lemma 1 and Lemma 2, one can prove the following existence theorem for the solution of (1) and (2). Since this theorem is analogous with the theorems [7, Thm 3], [8, Thm 2], [9, Thm 2], and the way of proving it is similar as well, we state it without a proof.

Theorem 1. *Let f be non-increasing, in the variable x, on R. Let there exist functions $\alpha, \beta \in C_3(I)$ being, respectively, a lower and an upper solution of the* BVP (1) *and* (2), *satisfying* (8). *Further, let there exist a positive constant L such that* (13) *holds.*

Finally, let α, β satisfy:

$$\alpha(a_1) \leqq k_1, \qquad k_2 \leqq \beta(a_1),$$

where k_1 and k_2 are the constants defined in (12); *at the same time, let the constants M_1 and M_2 satisfy*:

$$M_1 = \min_{\omega \times [-R_2, R_2]} f(t, x, x', x''), \qquad M_2 = \max_{\omega \times [-R_2, R_2]} f(t, x, x', x''),$$

where R_2 is the constant of Lemma 2, *with $R_2 \geqq |\alpha''(t)|, |\beta''(t)|$ on I.*

Then there exists at least one solution x of the BVP (1) *and* (2) *such that* (14) *is satisfied.*

Remark 1. The assumption of Theorem 1, requiring that f be non-increasing in x on R, can be omitted when the definition of lower and upper solutions of the BVP (1) and (2) is replaced by a new, stronger definition, obtained from the original one via replacing the first conditions in (6) and (7) by the following ones:

$$\alpha''' \geqq f(t, x, \alpha', \alpha''), \qquad \beta''' \leqq f(t, x, \beta', \beta'')$$
$$\text{for } t \in I \text{ and } x\colon \alpha(t) \leqq x \leqq \beta(t).$$

Remark 2. A similar existence theorem can be proved for the BVP (1) and (2), with the second condition in (2) changed to:

$$\beta_1 x(a_2) - \beta_2 x'(a_2) + \beta_3 x''(a_2) = A_2.$$

In this case, f is assumed to be non-decreasing in x. Moreover, α and β should satisfy: $\beta \leqq \alpha$, $\alpha' \leqq \beta'$, and for the values $\alpha(a_3)$ and $\beta(a_3)$ some restrictive conditions, implied by the estimates of the value in a_3 of the solution to the BVP for the modified equation, should be found.

3. A construction of lower and upper solutions

Theorem 2. *Let the function f have the following properties*:

(i) $f(t, x, x', x'') \geqq 0$ *on* $I \times R^3$.

(ii) *f is non-decreasing in the variable x on R, the other variables being fixed.*

(iii) *f is non-decreasing in the variable x' on R, the other variables being fixed.*

(iv) *There exists a positive constant L such that*

$$|f(t, x, x', x'') - f(t, x, x', y'')| \leqq L|x'' - y''|$$
$$\text{holds for } (t, x, x') \in I \times R^2,\ x'', y'' \in R.$$

Then there exist the functions α, $\beta \in C_3(I)$ *satisfying* (8) *and being, respectively, a lower and upper solution of the* BVP (1) *and* (2) *in the sense of Remark* 1.

Proof. 1. Let φ solve the BVP (3) and (2), and let c be an arbitrary non-negative constant. Then the assumption (i) implies that the function

$$\beta_c(t) = \varphi(t) + c$$

is an upper solution of the BVP (1) and (2) in the sense of Remark 1 under assumption that there exists some lower solution α and (8) holds.

Now let us fix the constant c, and consider the differential equation

$$x''' - L|x''| - K_c = 0\,, \tag{17}$$

where $K_c = \max_I (L|\varphi''(t)| + f(t, \varphi(t) + c, \varphi'(t), 0)$.

As the general solution x of this equation we obtain

$$x(t) = \begin{cases} c_1 + \left(c_2 - 2\dfrac{K_c}{L} t_0\right)t + \dfrac{K_c}{2L} t^2 - \dfrac{K_c}{L^3} e^{L(t_0 - t)}, & t \leqq t_0 \\ c_1 - \dfrac{2K_c}{L^3} - \dfrac{K_c}{L} t_0^2 + c_2 t - \dfrac{K_c}{2L} t^2 + \dfrac{K_c}{L^3} e^{L(t - t_0)}, & t \geqq t_0, \end{cases} \tag{18}$$

where t_0, c_1, c_2 are real constants.

From (18) one infers that there exists at least one solution $\Phi_c(t)$ of this equation, satisfying the following conditions:

$$\begin{aligned} \alpha_2 \Phi_c'(a_1) - \alpha_3 \Phi_c''(a_1) &\leqq 0 \ldots \text{(I)} \\ \beta_1 \Phi_c(a_2) + \beta_2 \Phi_c'(a_2) - \beta_3 \Phi_c''(a_2) &\leqq 0 \ldots \text{(II)} \\ \gamma_2 \Phi_c'(a_3) + \gamma_3 \Phi_c''(a_3) &\leqq 0, \ldots \text{(III)} \\ \Phi_c(t) \leqq 0\,, \quad \Phi_c'(t) \leqq 0\,, &\quad t \in I. \end{aligned} \tag{19}$$

Let us form the function

$$\alpha_c(t) = \varphi(t) + \Phi_c(t) \qquad \text{on } I.$$

The functions α_c and β_c satisfy the condition (8).

Let $t \in I$ and $x \leqq \beta_c(t)$. Then the assumptions (ii), (iii) and (iv) imply

$$\begin{aligned} f(t, x, \alpha_c', \alpha_c'') &\leqq f(t, \varphi + c, \varphi', \varphi'' + \Phi_c'') \leqq \\ &\leqq L|\Phi_c''| + L|\varphi''| + f(t, \varphi + c, \varphi', 0) \leqq \\ &\leqq L|\Phi_c''| + K_c = \Phi_c''' = \alpha_c'''. \end{aligned}$$

Therefore the functions α_c and β_c are, respectively, a lower and an upper solution of the BVP (1) and (2) in the sense of Remark 1.

4. Applications

In order to state briefly and prove the following theorems, we introduce some additional notation:

$$\omega_c = \{(t, x, x') \colon t \in I,\ \alpha_c(t) \leqq x \leqq \beta_c(t),\ \alpha_c'(t) \leqq x' \leqq \beta_c'(t)\},$$

where α_c and β_c are the functions constructed in the proof of Theorem 2,

R_{2c} = the constant of Lemma 2, corresponding to the functions α_c and β_c and such that $R_{2c} \geqq |\alpha_c''(t)|, |\beta_c''(t)|$ on I

$$M_{1c} = \min_{\omega_c \times [-R_{2c}, R_{2c}]} f(t, x, x', x'') \qquad M_{2c} = \max_{\omega_c \times [-R_{2c}, R_{2c}]} f(t, x, x', x''),$$

$$C_1 = -\frac{h}{\Delta}(\Delta_{11} + \Delta_{12}a_1 + \Delta_{13}a_1^2)\left(\frac{\alpha_2}{2}h + \alpha_3\right),$$

$$C_2 = \frac{1}{6}h_2^3 - \frac{\beta_2}{2\beta_1}h_2^2 - \frac{\beta_3}{\beta_1}h_2 - \frac{1}{6}h^3,$$

$$m_1 = \max_I |\varphi'(t)|, \qquad m_2 = \max_I |\varphi''(t)|,$$

$$m_3 = \max_I f(t, \varphi(t), \varphi''(t), 0), \qquad m_4 = \max_I f(t, \varphi(t), \varphi'(t), \varphi''(t)).$$

Theorem 3. *Let the function f satisfy the assumptions* (i)—(iv) *of Theorem* 2, *and let, moreover,*
(v) *there exist a constant c,* $0 < c < \infty$, *such that*

$$c > C_1 M_{2c}.$$

Then there exists at least one solution x of the BVP (1) *and* (2).

Proof. When using Theorem 1, keeping Remark 1 in mind, it suffices to find a pair of functions α_c and β_c, mentioned in the proof of Theorem 2, such that:

$$\alpha_c(a_1) \leqq \varphi(a_1) + C_1(M_{1c} + M_3) + C_2(M_{2c} + M_4),$$

$$\varphi(a_1) + C_1(M_{2c} + M_4) + C_2(M_{1c} + M_3) \leqq \beta_c(a_1).$$

Analogously with [7, Remark 1], the nonstrict inequaities can be replaced by the strict ones and the values M_3 and M_4 can be left out, under the conditions above. Moreover, (i) implies that the value M_{1c} may be replaced by zero. Then these conditions read as follows:

$$\alpha_c(a_1) < \varphi(a_1) + C_2 M_{2c}, \qquad \varphi(a_1) + C_1 M_{2c} < \beta_c(a_1). \tag{20}$$

By (v), it is immediate that there exists a pair of functions, α_c and β_c, such that the second inequality in (20) holds true.

Acording to (ii), (iii), for M_{2c} we have:

$$M_{2c} = \max_{I \times [-R_{2c}, R_{2c}]} f(t, \varphi(t) + c, \varphi'(t), x''). \tag{21}$$

Further, the same assumptions, Lemma 2 and the formula, (15), for calculating the constant R_{2c} imply that this constant does not depend on the values of the function α_c, but is, in fact, dependent on its first and second derivatives. Hence using (21), the same result is obtained for the constant M_{2c} as well. This means that the function Φ_c may be considered with a suitably small constant c_1 (cf. (18)), which can always be chosen in such a way that the first inequality in (20) is satisfied.

Theorem 4. *Let the function f satisfy the assumptions* (i)—(iv) *of theorem* 2 *and let, moreover, the following assumptions be satisfied*:

(vi) *There exists a positive constant* L_0 *such that*

$$|f(t, x, x', x'') - f(t, y, x', x'')| \leqq L_0|x - y|,$$
$$\text{for } (t, x', x'') \in I \times R^2, \ x, y \in R.$$

(vii) *For the constants* L_0 *and* L *the condition*

$$B_1 \overset{\text{def.}}{=} \frac{L_0 C_1}{Lh}\left((e^{Lh} - 1)(2L^2h\gamma_3 + 3Lh + 2L\gamma_3 + 2) - Lh(Lh - 1)\right) < 1 \tag{22}$$

is fulfilled.

Let $\gamma_2 = 1$, *in the boundary conditions* (2).

Then there exists at least one solution x of the BVP (1) *and* (2).

Proof. We show that when (vi) and (vii) are satisfied, then the same is true for the assumption (v) of Theorem 3, and this will make the proof complete.

Let c be a constant with the property $0 < c < \infty$. Let us search, to β_c, for a solution Φ_c of the equation (17) such that the conditions (19) are satisfied. Of course, this Φ_c is of the form (18), with $t_0 = a_1$:

$$\Phi_c(t) = c_1 - \frac{2K_c}{L^3} - \frac{K_c}{L}a_1^2 + c_2t - \frac{K_c}{2L}t^2 + \frac{K_c}{L^3}e^{L(t - a_1)} \tag{23}$$

Then $\Phi_c''(a_1) = 0$ and $\Phi_c''(t) > 0$ on (a_1, a_3), which implies that Φ_c' is increasing on I.

Now choose the constant c_2 so that $\Phi_c'(a_3) + \gamma_3\Phi_c''(a_3) = 0$. This means:

$$c_2 = \frac{K_c}{L}\left(a_3 - \frac{1}{L}e^{Lh} + \gamma_3 - \gamma_3 e^{Lh}\right).$$

When c_2 is so chosen, then $\Phi_c'(a_3) \leqq 0$ and therefore $\Phi_c'(a_1) < 0$. This yields that Φ', Φ'' satisfy the conditions (19)—(I), (III), and $\Phi_c'(t) \leqq 0$ on I.

(23) implies that there exists a sufficiently small constant c_1, such that the conditions (19)—(II) are also fulfilled, $\Phi_c(t) \leqq 0$ on I, and the first inequality of (20) is satisfied as well.

Let us choose the constant R_{1c}:

$$R_{1c} = \max_I |\varphi'(t)| + \max_I |\Phi_c'(t)|.$$

Then

$$R_{1c} = m_1 - \Phi_c'(a_1) = m_1 + \frac{K_c}{L}\left((e^{Lh} - 1)\left(\frac{1}{L} + \gamma_3\right) - h\right),$$

$$R_{1c} \geqq |\alpha_c'(t)|, \ |\beta_c'(t)| \text{ on } I.$$

Since

$$m_c = \max_{\omega_c}(L|\varphi''(t)| + f(t, x, x', \varphi''(t))) \leqq L_0 c + Lm_2 + m_4,$$

$$\max_I |\Phi_c''(t)| = \Phi_c''(a_3) = \frac{K_c}{L}(e^{Lh} - 1),$$

the constant R_{2c} can be chosen, thanks to the estimate (16), in the following form:

$$R_{2c} = 2LR_{1c} + h(L_0 + Lm_2 + m_4) + \frac{2R_{1c}}{h} + m_2 + \frac{K_c}{L}(e^{Lh} - 1).$$

Then

$$R_{2c} \geqq |\alpha_c''(t)|, \ |\beta_c''(t)| \qquad \text{on } I.$$

For the constant K_c one gets, according to its definition in (1.7), the estimate:

$$K_c \leqq L_0 c + Lm_2 + m_3, \tag{24}$$

and, using (21), for the constant M_{2c} one gets

$$M_{2c} \leqq L_0 c + LR_{2c} + m_3. \tag{25}$$

Using the results above, after inserting R_{2c} into (25), R_{1c} into R_{2c} and after using the inequality (24), we obtain:

$$C_1 M_{2c} \leqq B_0 + B_1 c,$$

where B_0 is a constant depending on the values α_i, β_i, γ_i, h, m_i, L_0 and L, and B_1 is the constant defined in (22).

By the assumption (vii), $B_1 < 1$, therefore there exists a constant c such that $0 < c < \infty$ and such that

$$c > B_0 + B_1 c \geqq C_1 M_{2c},$$

which implies that the assumption (v) of Theorem 3 is satisfied indeed.

Remark 3. Results similar to those obtained when $f \geqq 0$, can be derived when $f \leqq 0$.

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ПОСТРОЕНИЯ НИЖНИХ И ВЕРХНИХ РЕШЕНИЙ ДЛЯ НЕЛИНЕЙНОЙ КРАЕВОЙ ЗАДАЧИ ТРЕТЬЕГО ПОРЯДКА И ИХ ПРИМЕНЕНИЯ

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Резюме

В работе рассматривается трехточечная нелинейная краевая здача для обыкновенного дифференциального урвнения $x''' = f(t, x, x', x'')$ с линейными краевыми условиями. К этой задаче построены некоторые нижние и верхние решения для широкого класса функций f, которые применяются для составления теорем существования и решения краевой задачи.