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A NOTE ON RANDOMLY COMPLETE n–PARTITE GRAPHS

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ABSTRACT. The paper of Alavi et al. [1] contains two results concerning randomly complete n-partite graphs. We give improvements and a correction of their result.

Let G be a graph containing a subgraph H with no isolated vertices. In [2] the concept of "*randomly H graphs*" is introduced as follows: The graph G is said to be a randomly H graph if and only if any subgraph of G without isolated vertices, which is isomorphic to a subgraph of H, can be extended to a subgraph H_1 of G such that H_1 is isomorphic to H.

The authors [1] characterized graphs G that are randomly complete n-partite. Two of their results, namely, Theorem 1 and 2, are improved upon.

The first of them asserts:

THEOREM 1. [1]. *A graph G is randomly $K_{p,q}$, $q \ge p \ge 2$, if and only if G is isomorphic to a complete bipartite graph $K_{s,t}$, where $s \ge p$ and $t \ge q$, or G is isomorphic to a complete graph K_r where $r \ge p+q$.*

However, the graph $K_{3,5}$ is not randomly $K_{3,4}$. Otherwise, take F (Fig. 1.a), which is a subgraph of $K_{3,5}$. The graph F is isomorphic to a subgraph F' (Fig. 1.b) of $K_{3,4}$. But there is no way to extend F in $K_{3,5}$ to a graph isomorphic to $K_{3,4}$.

The result in Theorem 1 can be improved as follows:

THEOREM A. *A graph G is randomly $K_{p,q}$, $q \ge p \ge 2$, if and only if G is isomorphic to a complete graph K_r, where $r \ge p+q$ or*

(i) $p = q = 2$ *and* $G = K_{s,t}$, *where* $t \ge s \ge 2$,
(ii) $p = 2$, $q = 3$ *and* $G = K_{s,t}$, *where* $s \ge 2$, $t \ge 3$,
(iii) $p = 2$, $q \ge 4$ *and* $G = K_{s,t}$, *where* $s = 2$, $t \ge q$,

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(iv) $p = q = r \geq 3$ *and* $G = K_{r,r}$,
(v) $p = r \geq 3$, $q = r + 1$ *and* $G = K_{r,r+1}$ *or* $G = K_{r+1,r+1}$,
(vi) $p = r \geq 3$, $q = r + 2$ *and* $G = K_{p,q}$.

For the proof see [4, Theorem 2 and Theorem 3] (H-closed is there used for the term randomly H graph).

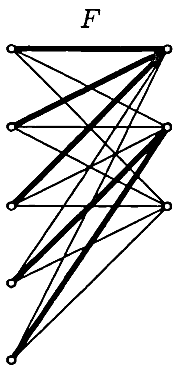

Figure 1.a.

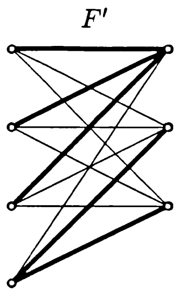

Figure 1.b.

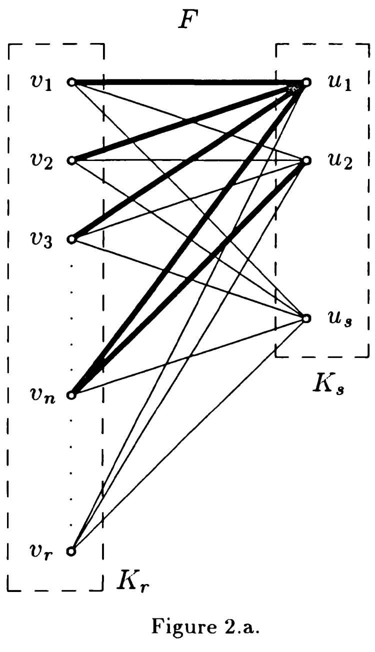

Figure 2.a.

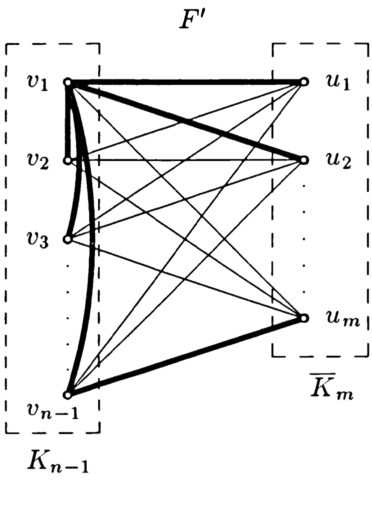

Figure 2.b.

The second result asserts:

THEOREM 2. [1]. *Let $m \geq 2$ and $n \geq 3$ be integers. The graph G is randomly $K_{n-1} + \overline{K}_m$ if and only if G is isomorphic to $K_r + \overline{K}_s$, where: $r \geq n-1$, $m \geq s \geq 0$, and $r + s \geq m + n - 1$.*

(The operation "+" is taken in the usual sense of Harary [3]).

The above assertion is true only in the case $m \leq 2$. For the case $m > 2$ we need the following:

PROPOSITION. *Let $m > 2$, and $n \geq 3$ be integers. Let $G = K_r + \overline{K}_s$, $H = K_{n-1} + \overline{K}_m$, $r > n-1$, $m \geq s \geq 2$, $r + s \geq m + n - 1$. The graph G is not randomly H graph.*

Proof. Assume that $G = K_r + \overline{K}_s$. Let

$$V_1 = \{v_1, v_2, \dots, v_n, \dots, v_r\} = V(K_r), \quad V_2 = \{u_1, u_2, \dots, u_s\} = V(\overline{K}_s)$$

be two disjoint sets of vertices of G. Form a graph F as follows: the edges of F are

$$E(F) = \{(u_1, v_1), (u_1, v_2), \dots, (u_1, v_n), (u_2, v_n)\},$$

(see Fig. 2.a). Obviously, F is a subgraph of $K_r + \overline{K}_s$, which is isomorphic to a subgraph F' of $K_{n-1} + \overline{K}_m$, (see Fig. 2.b). But there is no way to extend F in G to a subgraph that is isomorphic to $K_{n-1} + \overline{K}_m$.

And now, from this Proposition and from [5, Theorem 5] it follows:

THEOREM B. *Let $m = 2$ and $n \geq 3$. The graph G is randomly $K_{n-1} + \overline{K}_2$ if and only if G is isomorphic to $K_r + \overline{K}_s$, where $r \geq n-1$, $2 \geq s \geq 0$, $r + s \geq m + n - 1$.*

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