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# BOOK REVIEWS

Prékopa, A.: STOCHASTIC PROGRAMMING.
Akadémiai Kiadó, Budapest 1995, 599 pp.

The book of the well-known Hungarian mathematician is written as a textbook. The scope of solved problems is characterized by a brief characterization of single chapters. The best is to use the author's own words.

First three chapters are devoted to linear programming.

Chapter 4 is devoted to logconcave (logarithmically concave) probability measures and their extensions. Logconcave measures and their extensions provide us with the mathematical tools to establish the convexity of some of the stochastic programming problems, especially probabilistic constrained problems. Therefore, an in-depth presentation of the relevant most important theorems is given.

Chapter 5 presents fundamentals of optimization type moment problems. A large part of this chapter is devoted to discrete moment problems which came to prominence concerning bounding probabilities of Boolean functions of events and sets in higher dimensional spaces.

Chapter 6 pays more attention to these bounding procedures and presents probability approximation schemes, combining numerical integration and simulation.

Chapter 7 presents statistical decision theoretical principles in a historical framework.

Chapter 8 summarizes the basic static type stochastic programming model constructions.

Chapter 9 presents two solution techniques for the simple recourse problem for the case of discrete random variables. It is also shown how the case of the continuously distributed random variables can be treated.

Chapter 10 deals with the theorems concerning the convexity of probabilistic constrained stochastic programming problems. Many of them are derived from the results of Chapter 4.

Chapter 11 is devoted to programming under probabilistic constraints and the methodologically similar problems: maximizing a probability under constraints. More work has been done for the case of continuous random variables but at the end of the chapter a few results are mentioned for the case of the discrete distrubution. In this chapter, some nonlinear programming algorithms are presented which have been tested for the solution of probabilistic constrained problems.

Chapter 12 describes the basic fact concerning the model: two-stage programming under uncertainty, also called: stochastic programming with recourse.

Chapter 13 discusses the basic ideas about multi-stage stochastic programming problems.

In Chapter 14, a selection od some special applied problems is presented. Some of them are network type problems and a wide applicability for them may be expected.

Chapter 15 is devoted to distribution problems in stochastic programming. This area is rapidly developing, especially the analysis of stochastic combinatorial optimization problem is under intensive investigation. A sample also from these problems was taken.

The book is a nice well of information for everybody interested in stochastic programming however not only for him; many parts of the book are useful also for other domains of probability and matematical statistics (e.g., Chapters 1, 2, 7) and many parts are interesting themselves.

Many exercises and problems are given at the ends of the chapters. In addition, author also gives a recommendation how to prepare, by the help of the book, the master's and Ph.D. courses in stochastic programming.

The book should be available in each mathematical library (the price is about 85 USD).

*Lubomír Kubáček, Olomouc*

Meng, J.—Jun, Y. B.: BCK-ALGEBRAS.
Kyung Moon Sa Co., Seoul 1994, 294 + vi pp.
ISBN 89-7282-117-9

In 1966, Imai and Iséki [ImIs] introduced a notion of BCK-algebras. This notion originated from two different ways:

(1) set theory, and
(2) classical and non-classical propositional calculi.

In set theory, we have the following simple relations: $(A - B) - (A - C) \subseteq C - B$ and $A - (A - B) \subseteq B$.

Today BCK-algebras have been studied by many authors and they have been applied to many branches of mathematics, such as group, functional analyse, probability theory, topology, fuzzy set theory, and so on.

The monograph consists of eight chapters, appendix, and index.

Chapter I begins with basic BCK-algebra theory including several examples. We recall that a BCK-algebra is a non-empty set $X$ with a binary operation $*$ and with a constant element 0 satisfying the following axioms: for all $x, y, z \in X$ we have

(I) $\big((x * y) * (x * z)\big) * (z * y) = 0$;
(II) $\big(x * (x * y)\big) * y = 0$;
(III) $x * x = 0$;
(IV) $x * y = 0$ and $y * x = 0$ imply $x = y$; and
(V) $0 * x = 0$.

Then subalgebras, bounded BCK-algebras, positive implicative BCK-algebras, commutative BCK-algebras, BCK-algebras with condition (S) are introduced.

In Chapter II, the authors study ideals in BCK-algebras. They discussed ideals, implicative ideals, positive implicative ideals, commutative ideals, maximal ideals, Varlet ideals, prime and irreducible ideals, etc. In addition, quotient algebras are studied, too.

In Chapter III, homomorphisms and isomorphisms of BCK-algebras are presented. Some characterizations of obstinate ideals and homomorphism theorem are proved.

In the next chapter, the authors studied dual ideals, D-dual ideals, prime dual ideals. The relationship of dual ideals, D-dual ideals and filters, and the construction of the quotient algebra via a filter are presented.

In Chapters V and VI, some results on congruences and varieties of BCK-algebras are given. In particular, it is proved that BCK-algebras do not form a variety, this is an example of A. Wronski, 1983.

In Chapter VII, some structures of finite BCK-chain, subdirectly irreducible BCK-algebras, and the structures of semisipmle and simple BCK-algebras are studied. There are also introduced notions like atoms, radicals, stabilizers in BCK-algebras, and periodic BCK-algebras here.

In the final chapter, the authors study free, injective, projective and fuzzy BCK-algebras. Finally, the monograph is completed with complete classification tables of BCK-algebras with order $n \leq 5$ and its ideals.

The monograph is designed primarily for the graduate students who want to learn the basic ideas and techniques of the algebraic theory of BCK-algebras.

For me it is very surprising that practically nowhere in the monograph there is anything mentioned about MV-algebras. MV-algebras have a very close connection with BCK-algebras because D. Mundici in 1986 proved a very important fact that bounded commutative BCK-algebras are categorically equivalent to MV-algebras. Moreover, no measure theory on BCK-algebras is presented which gives a new area for the possible research in this direction.

Anyway, the monograph is very interesting and useful for any reader.

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Pap, E.: NULL-ADDITIVE SET FUNCTIONS.
Kluwer Academic Press, Dordrecht-Boston-London and
Ister Science, Bratislava 1995, 315 pp.
ISBN 0-7923-3658-5 (Kluwer Academic Publishers)
ISBN 80-88683-12-2 (Ister Science)

In 1953, G. Choquet was a pioneer in non-additive measure theory when he with his theory of capacities broke the hegemony of additive measure theory. Non-additive measures are, for example, outer measures, semi-variations of vector measures, they naturally appeared earlier in classical measure theory concerning $\sigma$-additive set functions. Non-additive set functions are intensively used in different parts of mathematics and they have applications in various branches of sciences and techniques.

We recall that by a null-additive set function we mean a mapping $\mu$ defined on a system of subsets $\mathcal{D}$ containing the empty set and closed under finite unions, and with values in the extended real axis such that $\mu(A \cup B) = \mu(A)$ whenever $A \cap B = \emptyset$ and $\mu(B) = 0$. The origins of null-additive set functions go back to V. N. Aleksjuk (as quasimeasures) in 1968, I. Dobrakov (as submeasures) in 1974 and L. Drewnowski in 1978. These functions were rediscovered by Wang in 1984 under the name null-additive set functions.

The monograph under the review is completely sacrificed to a systematical study of null-additive measures in various generalizations what concerns to the domain of definition of null-additive measures as well as to its range.

The book consists of twelve chapters starting with Introduction.

Chapter 2 – General Properties – defines basic notions related with null-additive functions and connections between them: measures based on triangular conorms, maxitive measures, fuzzy measures, submeasures, k-triangular measures, distorted measures.

In Chapters 3 and 4, three types of variations (disjoint, chain, and decomposition one) are introduced. In the next chapter, the autocontinuity of set functions is introduced. It presents a close relation between autocontinuous set functions with submeasures and some corresponding topologies (Frechet-Nikodym, one induced by submeasures). In addition, applications of families of submeasures to approximation theory are given, too.

Chapter 6 deals with decompositions of null-additive set functions. The author gives Hahn decomposition, Jordan decomposition, Lebesgue decomposition, Saks decomposition, and Hewitt–Yosida-type decomposition.

The most extended chapter of the book is Chapter 7 devoted to Choquet and Sugeno integrals. There are presented Choquet integrals, symmetric Choquet (Šipoš) integral, assymetric Choquet integral, and convergence-type theorems for these integrals.

Chapter 8 presents the latest results on integrals based on decomposable measures, i.e., measures defined on subsets of the real axis which are special kinds of semigroups. An interesting so-called $g$-calculus originally introduced by the author is given. This calculus can be applied, for example, to non-linear differential equations.

Chapter 9 studies ranges and regularities of null-additive set functions. There are presented generalizations of a Darboux-property theorem and Lyapunov-type theorems.

The following chapter is an attempt to the study of Radon–Nikodym type theorems, in particular, the case, when the universe $X$ consists of a finite number of elements, is studied.

Very interesting is the chapter devoted to k-triangular set functions where different versions of diagonal-type theorems for these set functions are presented. These results are important in the study of Nikodym convergence and boundedness type theorems and for Brooks–Jewett theorems.

The last chapter is devoted to convergence and boundedness theorems on difference posets which were introduced by Chovanec and Kôpka, students of the referee.

The monograph is completed by an extensive list of references and the index. I welcome this very interesting monograph which will be interesting to researchers and students in mathematics, as well as in such different fields as knowledge engineering, artificial intelligence, game theory, statistics, economics, sociology and industry.

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