Abstracts of theses in mathematics

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ABSTRACTS OF THESES* IN MATHEMATICS

defended recently at Charles University, Prague

SPECTRUM IN BANACH ALGEBRAS

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(March 11, 1996; supervisor V. Müller)

The spectrum is one of the most important concept in the theory of Banach algebras and Hilbert spaces. There are many generalizations of this concept. The Żelazko's axiomatic theory yields a unifying theory in case of spectra defined for commuting n-tuples of Banach algebra elements.

In this thesis a new axiomatic theory of spectrum is introduced (see V. Kordula, V. Müller, *On the axiomatic theory of spectrum*, Studia Math., to appear) and it is shown that it extends the Żelazko's axiomatic theory and includes also some well known spectra defined only for single elements of Banach algebras or operators (as the Apostol spectra or local spectra).

Some general results concerning this theory are proved, various types of spectra fitting to the theory are investigated and relations between these spectra are studied.

Finally, relations between spectra of two Banach algebra elements or Banach space operators under assumptions that there is some connection between them are investigated.

See V. Kordula, *The essential Apostol spectrum and finite dimensional perturbations*, Proc. Roy. Irish Acad., to appear;
V. Kordula, V. Müller, *The distance from the Apostol spectrum*, Proc. Amer. Math. Soc., in print;
V. Kordula, *On the defect spectrum of an extension of a Banach space operator*, Czech. Math. J., to appear.

CONTINUOUS AND ABSOLUTELY CONTINUOUS NORMS IN BANACH FUNCTION SPACES

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In this thesis the structure of weighted Banach function spaces and applications to the Hardy integral operator are studied. For the study of necessary and sufficient conditions for boundedness and compactness of the Hardy integral operator from a weighted Banach function space into L_∞ or BMO the notion of the "continuous norm in a weighted Banach function space" is used. (This notion

*An equivalent to PhD.

is more general than the well-known term of the "absolutely continuous norm in a weighted Banach function space" which was introduced in 1955 by W.A.J. Luxemburg.) Using this notion we show the difference between L_∞ and BMO as the target spaces of the Hardy integral operators. These results together with a new condition for compactness of the Hardy integral operator into BMO are contained in the first part of the thesis.

The next two parts of the thesis are devoted to the study of the difference between the notions of the "continuous norm" and the "absolutely continuous norm". Special weighted Banach function spaces are constructed to show all possible relations between subsets of functions with absolute continuous norms and subsets of functions with continuous norms. For example, we show that there exists a Banach function space such that its every element has a continuous norm while the only one element with the absolutely continuous norm is the zero function.

SEVERAL RESULTS RELATED TO THE ERDÖS-SZEKERES THEOREM

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The Erdős-Szekeres Theorem (1935) says that for any integer k there is an integer $n(k)$ such that any set of $n(k)$ points in the plane, no three on a line, contains k points which are vertices of a convex k-gon. In the thesis the following six results inspired by the Erdős-Szekeres Theorem are shown.

1. For any $d \geq 2$, there is an integer $h = h(d)$ such that there are arbitrarily large finite sets in general position in $\mathbb{R}^d$ with no h-hole. (For a finite set $A \subset \mathbb{R}^d$, a subset B, $B \subseteq A$, is called an *h-hole* if $|B| = h$ and the interior of $\operatorname{conv} B$ contains no point of A.)

2. There are n points in general position in the plane with less than $1.8n^2$ empty triangles (i.e., 3-holes), less than $2.42n^2$ empty quadrilaterals, less than $1.46n^2$ empty pentagons, and less than $n^2/3$ empty hexagons.

3. (Positive fraction Erdős-Szekeres Theorem) For any k there is a constant $c_k > 0$ such that any sufficiently large finite set $X \subset \mathbb{R}^2$ contains k subsets $Y_1, \ldots, Y_k$, each of size $\geq c_k|X|$, such that every set $\{y_1, \ldots, y_k\}$ with $y_i \in Y_i$ is in convex position.

4. Given a natural number k, let $rr(k)$ be the smallest integer such that, if n is sufficiently large with respect to k, and S is any set of n points in general position in the plane, then all but at most $rr(k)$ points of S can be partitioned into convex sets of sizes $\geq k$. Then $rr(k) = z(k)$, where $z(k)$ is the maximum size of a set S of points in general position in the plane with no convex k-set and with no convex sequence of length $k-2$.

5. For any finite set P of points in the plane and for any integer $k \geq 2$, there

is a finite set $R = R(P, k)$ with the following property: For any k-coloring of R there is a monochromatic set $\tilde{P}$, $\tilde{P} \subseteq R$, such that $\tilde{P}$ is combinatorially equivalent to the set P and the convex hull of $\tilde{P}$ contains no point of $R \setminus \tilde{P}$.

6. Any configuration combinatorially equivalent to a proper configuration in general position in the plane determines $l + 1$ distances whose ratios fall into prescribed intervals.

OPTIMIZATION PROBLEMS WITH INEXACT DATA

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The thesis deals with linear programming with inexact data — interval linear programming (ILP).

The criterion of strong unboundedness, the new necessary and sufficient condition for basis stability, and the new sufficient condition for basis stability of ILP problem are proved. Further, the strong feasible solutions of the interval linear programming problem with inequalities are investigated. The description of the set of strong solutions is given and the relationship between strong feasible solutions and strong feasibility is shown.

In the thesis the system of interval linear equations is investigated and new methods for computing the enclosure interval vector containing the exact interval solution of this system are found.

SETS OF DETERMINATION IN POTENTIAL THEORY

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In this thesis, sets of determination are studied. The first part gives a summary of known results on sets of determination in classical potential theory.

For a ball D (or a more general domain) in $\mathbb{R}^n$, a given strictly positive harmonic function h_0 and a class $\mathcal{H}$ of harmonic functions on D, a set M is called a set of determination with respect to $\mathcal{H}$ and h_0, if $\inf\limits_{x \in M} \frac{h(x)}{h_0(x)} = \inf\limits_{x \in D} \frac{h(x)}{h_0(x)}$ for all functions in $\mathcal{H}$.

The problem of characterizing sets of determination in the classical case has been studied by many authors (A. Beurling, B. Dahlberg, V.G. Mazja, F.F. Bonsall, W.K. Hayman, M. Essén, S. Gardiner, H. Aikawa ...).

In the rest of the thesis new results are contained. In the second part, sets of determination are characterized for parabolic functions, i.e. solutions of the heat equation, and for solutions of more general parabolic equations, on a slab. The situation is different as in view of the anisotropic character of the parabolic equation, only one-sided Harnack inequality is available. Sets of determination with

respect to 1 and a class of positive parabolic functions are described in terms of parabolic convergence and coparabolic (minimal) thinness; cfṘanošová J., *Sets of determination for parabolic functions on a half-space*, Comment. Math. Univ. Carolinae **35** (1994), 497–513; Ranošová J., *Characterization of sets of determination for parabolic functions on a slab by coparabolic (minimal) thinness*, Comment. Math. Univ. Carolinae **37** (1996), 707–723.

The case of the Helmholtz equation on $\mathbb{R}^n$ is considered, too; see Ranošová J., *Characterization of sets of determination for solutions of the Helmholtz equation*, to appear.

As an application certain decomposition theorems of functions belonging to $L^1(\mathbb{R}^n)$ are given.

In the last part, a relation between integral representations of bounded harmonic functions given by J. Bliedtner and sets of determination is studied using a selection theorem.

ROBUST METHODS IN THE LINEAR CALIBRATION MODEL

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(October 8, 1996; supervisor J. Jurečková)

The thesis introduces robust estimation and testing into the multivariate linear calibration model.

The history of the problem of calibration, the basic results achieved so far, and current applications are revived in Introduction, together with the formulation of the model and basic notation.

In Chapter I the linear model of multivariate controlled calibration is specified and the two stages of the calibration procedure described: calibration stage and prediction stage.

Chapter II is an introduction to M-estimators. Their definition, types and asymptotic properties are summarized for further reference.

Chapter III gives several options for a preliminary test of significance of parameters in the model using results derived earlier elsewhere in the literature.

Chapter IV deals with parameter estimation in the calibration stage, both with classical, regularization and robust techniques. Properties of particular estimators, exact and asymptotic, are given.

The estimation in the prediction stage is covered in Chapter V which is the fundamental part of the paper bringing original results. Asymptotic properties of the proposed estimator are treated here.

Chapter VI illustrates the subject matter of calibrating on two real data samples. Least squares and robust methods are compared on both clean and contaminated data.

M-ESTIMATORS IN LINEAR REGRESSION

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The thesis concerns various problems connected with M-estimators in linear models. It is divided into five parts. Chapter 1 is introductory.

In Chapter 2 some results concerning properties of M-estimators are summarized. Also robustness measures based on the influence function and on the change-of-variance function are derived.

The original results of the thesis are presented in Chapters 3 and 4. Chapter 3 consists of a theoretical section in which a dependence of some type of an M-estimator (e.g. Huber, Hampel, etc.) on a choice of its tunning constant is dealt with. This behaviour is analyzed from the point of view of its convergence to the model regression coefficients. In a numerical study a dependence of the Huber-estimator on a choice of its tunning constant is studied using some simulated data.

Chapter 4 deals with a problem when using several robust methods of estimation of regression coefficients we obtain for one set of data model estimates which values of regression coefficients are substantially different, and we have to select one of these estimates. In this chapter, a statistics which may be used to reject those estimates which are not adequate for given data is proposed. This statistics is based on the weighted Hellinger distance between kernel estimates of density of residuals in the two halves of the sample and the asymptotic distribution of this statistics is shown.