Acta Universitatis Carolinae. Mathematica et Physica

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Acta Universitatis Carolinae. Mathematica et Physica, Vol. 5 (1964), No. 2, 33--50

Persistent URL: http://dml.cz/dmlcz/142170

1964 ACTA UNIVERSITATIS CAROLINAE—MATHEMATICA ET PHYSICA 2 PAG. 33–50

Publications of the Astronomical Institute of the Charles University of Prague, No 41.

ATMOSPHERIC PRECIPITATION AND DIRECTLY MEASURED COMETARY CHARACTERISTICS

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Received on August 15, 1963

In the present study the period of variations is derived of the main cometary characteristics of the first class, called here the quantities of "cometary meteorology" owing to their physical significance, and the relations between these characteristics and the precipitation are investigated in West and Central Europe, as well as in the United States. The period of variations of the cometary characteristics is 6.3 or 6.2 years in the two areas, respectively. A high enough correlation has been found between the comet discoveries and the precipitation to show the higher precipitation the less comets are discovered. The long-term variations studied show the predominant influence of the comet hunters on the form of the smoothed curves. All the dependences found here have a purely statistical character.

1. INTRODUCTION

Two earlier papers of mine (SEKANINA 1959, 1960) have dealt with the variation of a number important physical characteristics of 563 comets in the course of the eleven-year solar cycle. The characteristics indicating a single wave, when plotted against the phase of solar cycle, are those directly measured and called the cometary characteristics of the first class. They are as follows:

(a) The number of comets, N_y, discovered per year regardless of the ephemeris.

(b) Yearly value of the function of the visual importance of cometary tails, τ_y; the function is defined as $\tau = 2$ if the tail is visible with the naked eye, $\tau = 1$ if the tail is visible by the telescope, and $\tau = 0$ if the comet has no tail.

(c) The average apparent magnitude of comets, m, at the time of discovery.

(d) The maximum angular diameter, D, of the atmospheres of comets.

(e) The maximum angular length, C, of the tails of comets.

The two latest quantities are, however, unknown for a number of comets and, hence, the main significance belongs to the first three characteristics. Since the directly observed characteristics are the question, they must be influenced by the conditions of visibility, first of all, by night cloudiness. The cometary discoveries, N_y, were considered the indicators of night cloudiness first by

LINK and VANÝSEK (1947) on the basis of the agreement of the N_y-curve with that of the precipitation in the course of the eleven-year cycle, the latter being studied by HELLMANN (1909). Hence the correlation between the night-cloudiness course and the precipitation course was postulated.

2. PROBLEM, AND AUXILIARY RESULTS

The verification of the hypothesis of the correlation between the cometary characteristics of the first class and the night-cloudiness activity must be based on the solution of three other problems:

(1) The precipitation variation during the eleven-year solar cycle, and such of its features as the problem of the commensurability of the precipitation period with the length of the solar cycle, the variation of the precipitation period, local differences, differences in the precipitation activity in the odd and even cycles, etc.

(2) The correlation between the precipitation amount and the night cloudiness.

(3) The correlation between the precipitation amount and the cometary characteristics of the first class.

Owing to the existence of the atmospheric circulation these problems cannot be studied all over the world simultaneously, but — strictly — for every spot on the Earth's surface separately. On the other hand, statistics of the cometary characteristics of the first class had to be investigated, of course, in a comparatively extensive area for the influence of the accidental errors on the form of resulting curves to be reduced as much as possible. Consequently, a compromise must be found between the requirements of meteorology on the one hand and of cometary statistics on the other. The frequency of the comet discoveries from the years 1610 till 1954 (Table 1) indicates that more than 50 per cent out of the entire number of comets have been discovered in West and Central Europe, while almost 25 per cent in North America, mostly in the United States. Hence the former of the two areas is first of all responsible for the resulting form of the curves of the cometary characteristics of the first class as found from the observational material. The latter of the two areas contributes to the form of the curves to some degree, too, while the other areas, in each of them not more than 8 to 9 per cent of comets have been discovered, may influence the curves in no way.

Table 1.

The distribution of the observational areas of cometary discoveries

area	percentage of comets
	%
West and Central Europe	50.3
North America	24.5
South Africa	8.7
East Europe	8.1
Far East	3.1
Australia and New Zealand	2.7
South America	2.3
Near and Middle East	0.3

My following paper (SEKANINA 1964) deals with the first of the three partial questions in the area of West and Central Europe, and briefly in the United States as well, the results of which we will make use of here.

The second of the three questions is far from to be solved completely at present. BOUŠKA (1950) analyzed the connection between the extension of night cloudiness and the total amount of precipitation in the northern part of Central Europe on the basis of the measures from 81 stations in the course of 1923 till 1933. He showed that both weather characteristics appear a strong positive correlation with each other. Some qualitative considerations concerning the correlation between night cloudiness and precipitation in West and Central Europe are included in Section 4.

Before we come to the third problem, an analysis of statistics of the cometary characteristics of the first class will be carried out from the same view-points as in the case of precipitation data (SEKANINA 1964).

3. CURVES OF THE QUANTITIES OF "COMETARY METEOROLOGY" IN WEST AND CENTRAL EUROPE

The characteristics given under (a), (b) and (c) in the Introduction, studied in West and Central Europe and reduced to yearly values by averaging three neighbouring yearly data, will be here called the quantities of "cometary meteorology" in West and Central Europe. The respective curves

$$N_y = N_y(t), \quad \tau_y = \tau_y(t), \quad m = m(t)$$

comprise both the short-term and the long-term variations in time. Since the influence of the latter is comparatively considerable, they have to be removed by introducing the time course of the functions

$$N_y^{(o)} = N_y^{(o)}(t), \quad \tau_y^{(o)} = \tau_y^{(o)}(t), \quad m^{(o)} = m^{(o)}(t),$$

obtained by smoothing out 25 neighbouring values of the initial curves. The resulting quantities, being free of the long-term variations according to the relations

$$\left.\begin{aligned} \Delta N_y &= N_y - N_y^{(o)}, \\ \Delta\tau_y &= \tau_y - \tau_y^{(o)}, \\ \Delta m &= -\{m - m^{(o)}\}, \end{aligned}\right\} \tag{1}$$

are in Fig. 1 represented by the second, third and fourth curves, respectively. For the sake of comparison, the integrated precipitation course, ΔA, is included at the top of Fig. 1.

Now let us study the variations of ΔN_y, $\Delta\tau_y$ and Δm. If t_k and t_l are the moments of the minima and maxima on the curves of the quantities of "cometary meteorology", respectively, and m, n their respective numbers, the period P of their variations is

$$P = \frac{1}{m+n-2}\left\{\sum_{k=1}^{m}(t_k - t_{k-1}) + \sum_{l=1}^{n}(t_l - t_{l-1})\right\}, \tag{2}$$

while the formulae for establishing the mean time of the minima and that of the maxima are

$$T_0\,(\min) = \frac{1}{m}\sum_{k=1}^{m} t_k \pm P\left(m_0 + \frac{m-1}{2}\right) \tag{3}$$

and

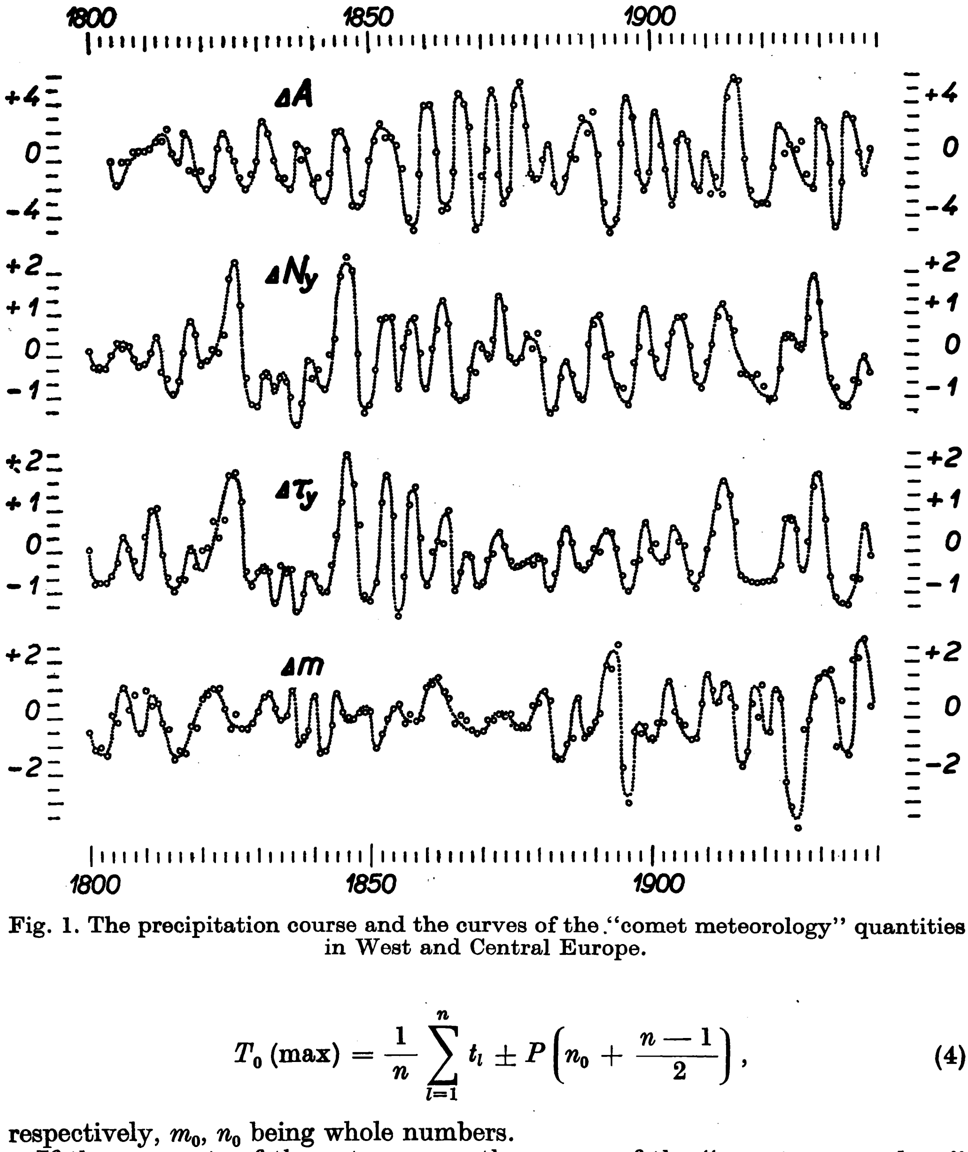

Fig. 1. The precipitation course and the curves of the "comet meteorology" quantities in West and Central Europe.

$$T_0\,(\max) = \frac{1}{n}\sum_{l=1}^{n} t_l \pm P\left(n_0 + \frac{n-1}{2}\right), \tag{4}$$

respectively, m_0, n_0 being whole numbers.

If the moments of the extremes on the curves of the "comet meteorology" quantities are established graphically, formulae (2) to (4) yield the results included in Table 2. The latest line gives the phase distance of the time of the maximum of the curve relative to that of its minimum. The correlation coefficient $\psi^*(o, e)$ between the courses of the "comet meteorology" quantities in the odd and even cycles of solar activity are for each of them presented in Table 3.

Analogously to the precipitation, the "comet meteorology" quantities appear the variation in the length of the period as well, within the limits of 4 to 12 years. The length of the period as a function of time is in connection with the

Table 2.

Time parameters of the curves of the "comet meteorology" quantities in West and Central Europe

	ΔN_y	$\Delta\tau_y$	Δm
P_0	6.29 ± 0.18	6.29 ± 0.19	6.29 ± 0.25
T_0 (min)	1906.02 ± 0.46	1905.98 ± 0.47	1905.73 ± 0.41
T_0 (max)	1902.96 ± 0.50	1902.87 ± 0.49	1902.71 ± 0.43
φ_0	0.51 ± 0.04	0.51 ± 0.05	0.52 ± 0.06

Table 3.

The correlation degree $\psi^*(o, e)$ of the "comet meteorology" quantities in West and Central Europe

Quantity	$\psi^*(o, e)$
ΔN_y	+0.06
$\Delta\tau_y$	+0.17
Δm	+0.15

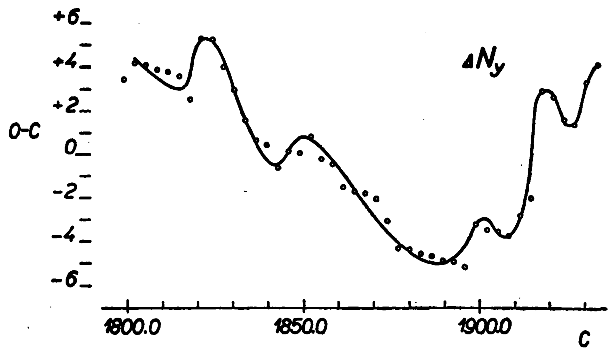

Fig. 2. The period variations of the curve of comet discoveries in West and Central Europe.

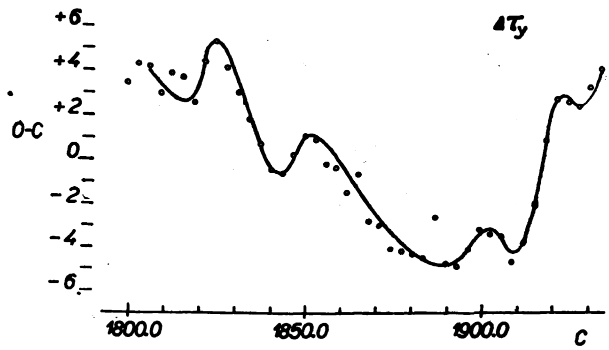

Fig. 3. The period variations of the curve of the function of the visual importance of cometary tails in West and Central Europe.

tangent to the $(O - C)$-curve, the latter being the difference between the observed and computed moments of the extremes. Figs. 2 to 4 show the $(O - C)$-curve for ΔN_y, $\Delta\tau_y$ and Δm, respectively. When comparing these curves with the precipitation $(O - C)$-curve (Fig. 5) we find that there is no resemblance between any out of the "comet meteorology" curves and the precipitation curve in the period of 1820 till 1940. The $(O - C)$-curve for Δm indicates a period of about 60 years, while for ΔN_y and $\Delta\tau_y$ the long-term variations are beyond any doubt longer, more than 100 years, a few fluctuations being superimposed of much smaller amplitudes, the actual existence of which is questionable.

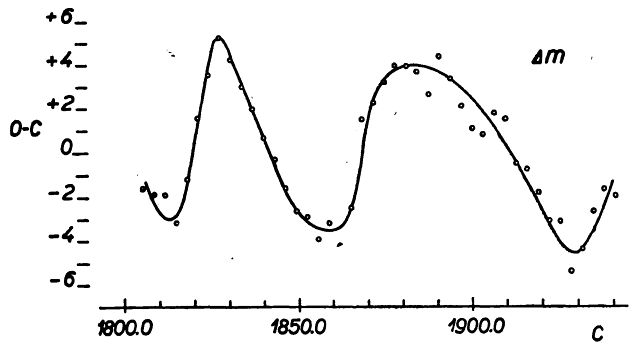

Fig. 4. The period variations of the curve of the apparent brightness of comets at the time of discovery in West and Central Europe.

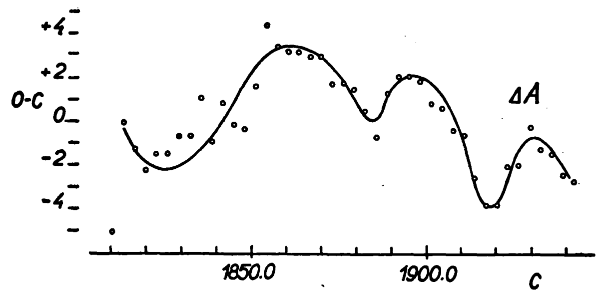

Fig. 5. The period variations of the precipitation activity in West and Central Europe.

4. CURVES OF THE "COMET METEOROLOGY" QUANTITIES AS RELATED TO THE PRECIPITATION-ACTIVITY VARIATIONS IN WEST AND CENTRAL EUROPE. CORRELATION COEFFICIENTS AND PHASE SHIFTS

Two main problems are endeavoured to be solved here as follows:

(a) Verification of some features of the mutual connections between the "comet meteorology" quantities, having been found to be the relations between the cometary characteristics of the first class in my earlier paper (SEKANINA 1960).

(b) Discussion of the hypothesis pronounced by LINK and VANÝSEK (1947), concerning the nature of the connection of the comet discoveries, or the "comet meteorology" quantities on the general, with the precipitation, assuming the latter being in a positive correlation with night cloudiness.

From Table 2 it follows immediately that on the average, i. e. regardless of the variations in the period length of the "comet meteorology" curves, the equal extremes are close to one another when compared the three curves with each other. Hence, we can assert:

Regardless of the period variations in the "comet meteorology" quantities, the time of the maximum frequency of comet discoveries corresponds to that of the maximum value of the function of the visual importance of comet tails and to that of the maximum brightness of comets at the time of discovery, and vice versa.

This assertion is in full agreement with what was found in Sections 12 and 13 of the above-mentioned paper (SEKANINA 1960) on the basis of the study of the differences of the so-called cycle values of the cometary characteristics of the first class in odd and even cycles of solar activity.

Another assertion may be pronounced as regards the latter problem, when comparing the results of Table 2 with those of Section 5 of my following study (SEKANINA 1964):

Regardless of the period variations in the precipitation curve and in the curves of the "comet meteorology" quantities, the time of the maximum precipitation follow the time of the minimum frequency of comet discoveries and that of the two other equivalent magnitudes, with about a yearly retardation, and vice versa.

This assertion corroborates both the hypothesis pronounced by LINK and VANÝSEK, and the well-founded introduction of the so-called subjective factor in Section 13 of my paper (SEKANINA 1960).

Nevertheless, in studying the behaviour of all the four curves of Fig. 1 in various periods, the two assertions lose their validity because of the period variations. The phase shift between any two out of the four curves ΔN_v, $\Delta \tau_v$, Δm, ΔA, is, hence, a function of time, and this effect must appear in time variability of the correlation degree.

First of all, an expression is derived for the dependence of the correlation coefficient on the phase shift, $\Delta\,\Phi$, of two functions, y_1, y_2, given by the cosine curves as follows:

$$\left.\begin{aligned} y_1 &= Y_1 \cos \frac{2\pi}{k}\,(\Phi - \Phi_0), \\ y_2 &= Y_2 \cos \frac{2\pi}{k}\,(\Phi - \Phi_0 + \Delta\Phi), \end{aligned}\right\} \quad (5)$$

where $k = \dfrac{P}{P_\odot}$, P is the period of the "comet meteorology" curves, or of the precipitation curve, $P_\odot$ is the length of the solar cycle. If the formula (20) of the following paper (SEKANINA 1964) is applied to the functions of (5), we obtain after the integration:

$$\left.\begin{aligned} \psi(\Delta\Phi) = E(k) \cos \frac{2\pi}{k}\Delta\Phi \,.\left[\mu_0 + \mu_1 \cos \frac{4\pi}{k}\,\Delta\Phi + \mu_2 \sin \frac{4\pi}{k}\,\Delta\Phi\right] - \\ - \frac{k}{2\pi}\,E(k) \sin \frac{2\pi}{k}\Delta\Phi \,.\left[\nu_0 + \nu_1 \cos \frac{4\pi}{k}\,\Delta\Phi + \nu_2 \sin \frac{4\pi}{k}\,\Delta\Phi\right], \end{aligned}\right\} \quad (6)$$

where

$$\left.\begin{aligned} E(k) &= 1 - \frac{k}{4\pi} \sin \frac{2\pi}{k} \left(\cos \frac{2\pi}{k} - \frac{k}{\pi} \sin \frac{2\pi}{k}\right), \\ F(k) &= 1 + \frac{k}{2\pi} \sin \frac{2\pi}{k} \left(\cos \frac{2\pi}{k} - \frac{k}{\pi} \sin \frac{2\pi}{k}\right), \\ G(k) &= \sin \frac{2\pi}{k} + \frac{k}{\pi} \left(\cos \frac{2\pi}{k} - 1\right), \end{aligned}\right\} \quad (7)$$

and

$$
\left.
\begin{aligned}
\mu_0 &= \left(1 + \frac{k^2}{4\pi^2}\right) F(k), \\
\mu_1 &= -\frac{k}{4\pi} \cos \frac{2\pi}{k} \left[G(k) - \frac{k}{\pi}\right] . F(k), \\
\mu_2 &= \frac{k}{4\pi} G(k) \sin \frac{2\pi}{k} \left[F(k) - \frac{k^2}{2\pi^2} \cos \frac{2\pi}{k}\right], \\
\nu_0 &= G(k) \sin \frac{2\pi}{k} \left[1 + \frac{k^2}{4\pi^2} \left(1 - 2 \cos \frac{2\pi}{k}\right)\right], \\
\nu_1 &= -\frac{k}{8\pi} \sin \frac{4\pi}{k} G(k) \left[G(k) - \frac{k}{\pi}\right], \\
\nu_2 &= \frac{k}{2\pi} \left[\cos \frac{2\pi}{k} F(k) + \frac{1}{2} \sin^2 \frac{2\pi}{k} G^2(k)\right].
\end{aligned}
\right\} \quad (8)
$$

The $\Delta\Phi$-shift of equations (5) and (6) must be understood in its absolute value. For $k = 0.563$, corresponding to the period of 6.2 to 6.3 years we have:

$$
\begin{aligned}
E &= +1.0151, \\
F &= +0.9699, \\
G &= -1.1363, \\
\mu_0 &= +0.9777, \\
\mu_1 &= +0.0094, \\
\mu_2 &= +0.0486, \\
\nu_0 &= +1.1270, \\
\nu_1 &= +0.0108, \\
\nu_2 &= +0.0705.
\end{aligned}
$$

The dependence of the correlation coefficient $\psi(\Delta\Phi)$ on the phase shift is for the given period, P, represented in Fig. 6 by a thin continuous curve. As seen the mutual shift of the cosine curves of about 1.5 year yields a zero correlation degree.

Table 4.

The correlation coefficients between the time courses of the cometary characteristics in individual solar cycles in West and Central Europe

cycle number	$\psi(\Delta N_y, \Delta\tau_y)$	$\psi(\Delta N_y, \Delta m)$	$\psi(\Delta\tau_y, \Delta m)$
7	$+0.954 \pm 0.018$	(-0.289 ± 0.187)	(-0.309 ± 0.184)
8	$+0.753 \pm 0.092$	$+0.118 \pm 0.210$	$+0.630 \pm 0.129$
9	$+0.920 \pm 0.030$	-0.014 ± 0.195	-0.032 ± 0.195
10	$+0.753 \pm 0.088$	$+0.226 \pm 0.193$	-0.024 ± 0.203
11	$+0.546 \pm 0.137$	$+0.463 \pm 0.153$	$+0.775 \pm 0.078$
12	$+0.483 \pm 0.156$	$+0.216 \pm 0.194$	-0.414 ± 0.168
13	$+0.671 \pm 0.107$	$+0.220 \pm 0.185$	$+0.722 \pm 0.093$
14	$+0.803 \pm 0.069$	$+0.162 \pm 0.190$	$+0.529 \pm 0.140$
15	$+0.938 \pm 0.026$	$+0.123 \pm 0.210$	$+0.226 \pm 0.202$
16	$+0.871 \pm 0.051$	$+0.209 \pm 0.204$	$+0.075 \pm 0.212$

Table 5.

The correlation coefficients between the time courses of the precipitation and of the cometary characteristics in individual solar cycles in West and Central Europe

cycle number	$\psi(\Delta A, \Delta N_y)$	$\psi(\Delta A, \Delta\tau_y)$	$\psi(\Delta A, \Delta m)$
7	$+0.027 \pm 0.203$	$+0.040 \pm 0.203$	$(+0.710 \pm 0.101)$
8	-0.207 ± 0.204	-0.368 ± 0.184	-0.283 ± 0.197
9	$+0.154 \pm 0.190$	$+0.088 \pm 0.193$	-0.085 ± 0.194
10	-0.729 ± 0.095	-0.709 ± 0.101	$+0.344 \pm 0.179$
11	-0.291 ± 0.178	-0.103 ± 0.193	-0.094 ± 0.193
12	-0.205 ± 0.195	-0.426 ± 0.167	$+0.205 \pm 0.195$
13	-0.148 ± 0.191	-0.616 ± 0.121	-0.761 ± 0.082
14	$+0.051 \pm 0.194$	-0.329 ± 0.174	-0.376 ± 0.167
15	$+0.714 \pm 0.105$	$+0.691 \pm 0.111$	-0.237 ± 0.201
16	$+0.285 \pm 0.196$	$+0.510 \pm 0.158$	$+0.063 \pm 0.213$

Now the two problems outlined at the beginning of this section may be solved in detail. In Table 4 the correlation coefficients are included between any two out of the three cometary characteristics, ΔN_v, $\Delta\tau_v$, Δm, in the period of the years 1823 till 1934, i. e. in the course of ten solar cycles, while Table 5 those between the precipitation and each of the three cometary characteristics in the same period.

To be able to compare the theoretical relation (6) with the empirically ascertained values we must still determine the mean value of the mutual phase shift, $\Delta\Phi$, of both curves for all the combinations of Tables 4 and 5 and for each solar cycle. Let us denote X_1, X_2 two out of the four quantities ΔN_v, $\Delta\tau_v$, Δm, ΔA, further $C(X_i)$ the computed moments of the extremes of the curves, included in Table 2 for the "comet meteorology" quantities and taken over from the following paper (SEKANINA 1964) for the precipitation, and $(O - C)_{X_i}$ the function given in Figs. 2 to 5. Then

$$\Delta\Phi = \frac{1}{P_\odot} \{[C(X_1) - C(X_2)] + (O - C)_{X_1} - (O - C)_{X_2}\} + n_0 k, \qquad (9)$$

$n_0 = 0, \pm 1$, k is again the reduced period.

The empirical values, representing the dependence of the correlation coefficient on the phase shift of the investigated curves, are given in Fig. 6. Let be pointed out that, firstly, $\Delta\Phi$ is understood as a phase difference between the moments of the equal extremes when the relations $(\Delta N_v, \Delta\tau_v)$, $(\Delta N_v, \Delta m)$ and $(\Delta\tau_v, \Delta m)$ are studied (an upper sign at the ordinate data of Fig. 6 is valid for the correlation coefficient), and as a phase difference between the moments of the opposite extremes for the relations $(\Delta A, \Delta N_v)$, $(\Delta A, \Delta\tau_v)$ and $(\Delta A, \Delta m)$ (a bottom sign is valid), and secondly, due to an insufficient number of comets discovered in West and Central Europe in the 7th solar cycle the time course of Δm is derived very uncertainly and, hence, none out of the correlation coefficients describing the relations $(\Delta A, \Delta m)$, $(\Delta N_v, \Delta m)$ and $(\Delta\tau_v, \Delta m)$ is plotted in Fig. 6.

Since a number of other, mathematically inexpressible factors reduce the empirically found correlation degree, instead of the "theoretical" correlation coefficient, given by (6) and written in an approximate form (for $k = 0.563$) as

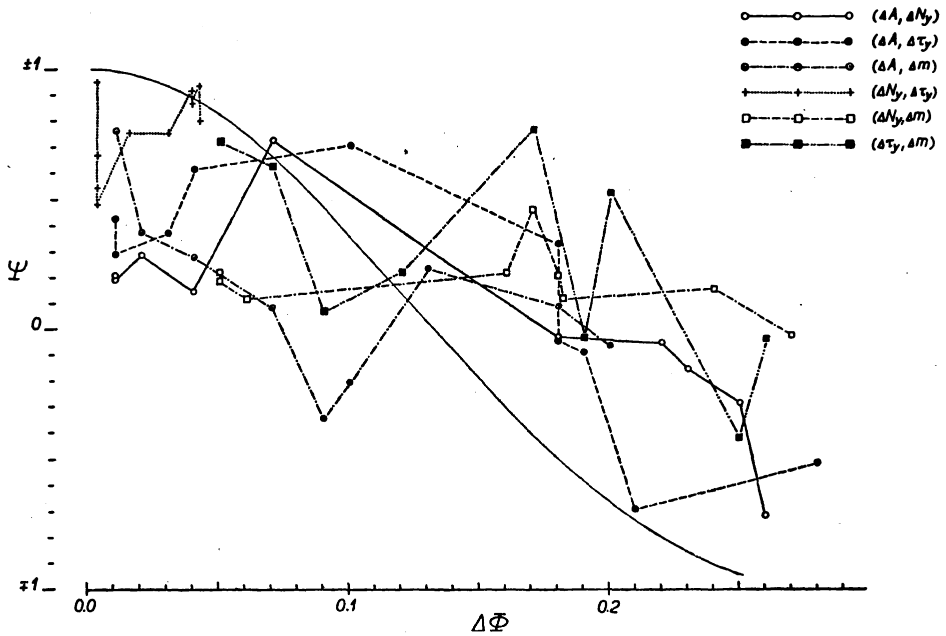

Fig. 6. A comparison of the theoretical form with the empirical data of the dependence of the correlation coefficient on the phase shift for the curves of the "comet meteorology" quantities and for the precipitation in West and Central Europe.

$$\psi(\varDelta\Phi) = \cos\frac{2\pi}{k}\varDelta\Phi - 0.098\sin\frac{2\pi}{k}\varDelta\Phi, \tag{10}$$

we introduce the correlation index $\psi_0(\varDelta\Phi)$, defined by the relation

$$\psi_0(\varDelta\Phi) = \psi_0(0)\left(\cos\frac{2\pi}{k}\varDelta\Phi - 0.098\sin\frac{2\pi}{k}\varDelta\Phi\right), \tag{11}$$

$\psi_0(0)$ is the reduced correlation index. The curve of $\psi_0(\varDelta\Phi)$ has the only common point with that of $\psi(\varDelta\Phi)$ for $\varDelta\Phi = 0.132$, where $\psi_0 = \psi = 0$. Equation (11) is a simple generalization of the relation between the correlation coefficient and the phase shift of two curves for the case that $|\psi(0)| < 1$. There is no doubt that index $\psi_0(0)$ is in this case very close to coefficient $\psi(0)$, but numerically it can be computed much easier. Its properties are analogous to those of the correlation coefficient. Then, ψ_0 on the left side of (11) being replaced by correlation coefficient ψ_m derived from the material (see Tables 4 and 5) leads to the following expression for the reduced correlation index of each of the six curves of Fig. 6:

$$\psi_0(0) = \frac{\sum\psi_m\left(\cos\frac{2\pi}{k}\varDelta\Phi - 0.098\sin\frac{2\pi}{k}\varDelta\Phi\right)}{\sum\left(\cos\frac{2\pi}{k}\varDelta\Phi - 0.098\sin\frac{2\pi}{k}\varDelta\Phi\right)^2}, \tag{12}$$

where we sum up over all the solar cycles included in the observational material.

Table 6.

Values of the reduced index of correlation between the cometary characteristics in West and Central Europe

correlation	$\psi_0(0)$	$\frac{\psi_0(0)}{\Delta\psi_0(0)}$	$\overline{\Delta\Phi}$	cycles
$(\Delta N_v, \Delta\tau_v)$	$+0.806 \pm 0.044$	18.3	$+0.006$	7—16
$(\Delta N_v, \Delta m)$	-0.025 ± 0.077	0.3	$+0.024$	8—16
$(\Delta\tau_v, \Delta m)$	$+0.207 \pm 0.159$	1.3	$+0.018$	8—16

Table 7.

Values of the reduced index of correlation between the precipitation and the cometary characteristics in West and Central Europe

correlation	$\psi_0(0)$	$\frac{\psi_0(0)}{\Delta\psi_0(0)}$	$\overline{\Delta\Phi}$	cycles
$(\Delta A, \Delta N_v)$	-0.325 ± 0.061	5.3	$+0.077$	7—16
$(\Delta A, \Delta\tau_v)$	-0.424 ± 0.090	4.7	$+0.083$	7—16
$(\Delta A, \Delta m)$	-0.287 ± 0.097	3.0	$+0.102$	8—16

Index $\psi_0(0)$ characterizes thus the correlation degree in the case of no phase shift between both investigated curves. The value of $1 - \psi_0(0)$ goes on the fluctuations in the size of amplitudes and on the deviations of the actual curves from the assumed cosine curves.

The resulting values of the correlation index, $\psi_0(0)$, are presented in Tables 6 and 7 for the relations between the "comet meteorology" quantities and for those between the precipitation and any of the "comet meteorology" quantities, respectively. Moreover, these tables give the ratio between the value and its probable error of the correlation index, and the phase distance of the equal extremes in Table 6, and those of the opposite extremes in Table 7. The sign of $\overline{\Delta\Phi}$ is positive, if the phase of the curve of the first mentioned quantity is greater, and vice versa. Finally, the latest column lists the solar cycles, for which the given value of $\psi_0(0)$ is derived.

The following conclusions can be arrived at on the basis of the data of Tables 6 and 7:

(1) The cometary discoveries, ΔN_v, and the function of the visual importance of cometary tails, $\Delta\tau_v$, appear a minute mutual shift and a high degree of correlation when reduced to the equal moments of the extremes.

(2) On the other hand the relation between the apparent magnitude at the time of discovery, Δm, and the cometary discoveries (or the tail function) shows after reduction such a little degree of correlation that it cannot be considered real; the mutual shift amounts to about 3 months on the average.

(3) All the three quantities of "cometary meteorology" further seem to indicate real correlations with the precipitation, ΔA. The correlations of the precipitation with the cometary discoveries and of the precipitation with the tail function is real beyond any doubt, while the connection between the precipitation and the apparent brightness is less conspicuous, nevertheless, it

exists. The mutual average shifts of the corresponding (i. e. opposite) extremes are now much greater. The precipitation follows the curves of the "comet meteorology" quantities for about 10 to 13 months.

(4) After the phase shift of the curves is reduced, the value of the correlation degree, given as a ratio between the reduced correlation index and its error, as well as the value of the average phase shift of the corresponding extremes on the curves are in close connection with one another both in the case of the mutual relations among the quantities of "cometary meteorology", and in the case of the relations between the precipitation activity on the one hand and the "comet meteorology" quantities on the other. In the two cases the numerical parameters of the relation are, of course, different from each other. In the case of the correlations concerning only the quantities of "cometary meteorology" the phase shift of $+0.01\ P_\odot$ yields a change in the correlation degree $\frac{\psi_0(0)}{\Delta\psi\ (0)}$ of —12 units, while in the case of the correlations concerning the precipitation as well, the same phase-shift change yields a change of only —1 unit in the correlation degree. It means that even the mean values of the moments of the extremes computed from formulae (3) and (4) may be considered a criterion.

In these assertions a fact is proved on the basis of objective methods that by means of the reduction to the zero phase shift between the extremes of precipitation and those of the "comet meteorology" curves a negative reduced correlation index yields, which indicates that the minimum of comet discoveries, the minimum on the curve of the tail function, and the minimum brightness of comets at the time of discovery correspond to the maximum precipitation, and vice versa.

The distribution of the phase shifts of the extremes of each of the three curves of the "comet meteorology" quantities relative to the nearest opposite precipitation extreme is plotted in Fig. 7, where we can see that in about 40 per cent out of all the cases the phase shifts are less than ± 4 months and in 75 per cent less than ± 2 years. Hence, this criterion leads also to the conclusion that, statistically, the minima of the "comet meteorology" quantities fall on the precipitation maxima. Some individual great phase shifts have not to be explained by some artifical considerations, even when they can be produced by the increase of the influence of certain atmospheric processes.

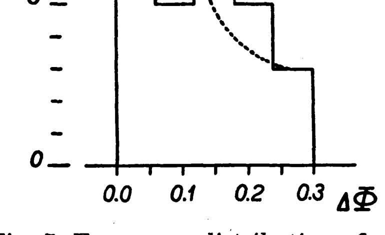

Fig. 7. Frequency distribution of the phase shifts of the extremes of the curves of the "comet meteorology" quantities relative to the precipitation in West and Central Europe.

For instance, some systematic deviations probably exist from the linear dependence between night cloudiness and the precipitation that have not been taken into consideration. A very good agreement between night cloudiness and the precipitation may be expected in the European Oceanic area, where the precipitation is in the course of 24 hours distributed roughly uniformly; on the other hand, in the Central European area the precipitation in the summer period is in connection with the cloudiness of the convective character, which is extensive by day, but it dissolves by night. It would be interesting to investigate the statistical material from this point of view. Up to now, we must refer only to the mentioned paper by BOUŠKA (1950), which leads to the positive correlation.

However, great phase shifts of the accidental character may be explained, recalling a result of my following paper, concerning the eastwards phase retardation of the precipitation activity over the Continent, amounting to about 1.4 km per day. For example, a shift, relative to the centre of the precipitation area considered, of the area centre of the activity of comet hunters in the course of a solar cycle for 5 degrees in the geographical longitude yields a phase shift between the comet-discoveries curve and the precipitation curve of about 0.7 years. Since the precipitation area has an average dispersion in the geographical longitude of about 10 degrees, a phase-shift dispersion of 1.4 year does not yet contradict the statistical validity of the relation between the precipitation activity and the "comet meteorology" quantities. As seen the considerable inertia of the mechanism, controlling the meteorological situation over West and Central Europe, is a very conspicuous factor, which must all the time be kept in view, when analyzing the investigated relation.

Summarizing the results of the study of both basic problems submitted at the beginning of this section, we must emphasize:

(a) A high degree of correlation is confirmed between the two basic members of the first class of cometary characteristics, namely the cometary discoveries, N_y, and the tail function, τ_y, in West and Central Europe, which was earlier discovered both in the course of the eleven-year cycle and in the relation between the cycle-values of the two quantities in odd and even cycles (SEKANINA 1960). Furthermore, the apparent brightness of comets at the time of discovery is confirmed to behave distinctly from both just mentioned characteristics in short intervals of time, which is expressed through a little correlation degree in Table 6. At the same time, however, the time parameters of the apparent brightness have been found to be close to those obtained for the two former quantities of "cometary meteorology" (Table 2). An analogous fact has been found in Sections 12 and 13 of my earlier paper (SEKANINA 1960), where the behaviour of the cometary characteristics is studied in the course of odd and even solar cycles.

(b) As regards the relation between the precipitation activity on the one hand and the quantities of "cometary meteorology" on the other, correctness of the hypothesis by LINK and VANÝSEK (1947) seems to be confirmed in the sense that the latter may be understood as indicators of night cloudiness, so that this hypothesis may be applied in statistical considerations and methods. The cometary discoveries and then the function of the visual importance of cometary tails result as the most realiable characteristics of the precipitation over West and Central Europe, while the apparent brightness of comets at the time of discovery seems to be less reliable, which may be produced by a number of effects, as e. g. the inaccuracy of initial data in the observational material, the considerable dispersion in the size of the amplitude of curve $\Delta m = \Delta m(t)$ etc. Simultaneously, the justification is obvious of applying the term of "quantity of 'cometary meteorology'" to the characteristics N_y, τ_y and m, the term being introduced more or less a priori at the beginning of this paper.

5. LONG-TERM VARIATIONS IN THE "COMET METEOROLOGY" QUANTITIES IN WEST AND CENTRAL EUROPE

The long-term variations in the "comet meteorology" quantities, i. e. the smoothed out quantities $N_y^{(o)}$, $\tau_y^{(o)}$ and $m^{(o)}$ differ as for their character essentialy from the quantities ΔN_y, $\Delta \tau_y$ and Δm, since the former comprise "subjective" factors, i. e. the influence of the instrumental equipment of observers and mainly the existence of the comet hunters, in addition to the "objective" long-term factors. The influence of the "subjective" factors on the deformation of the "objective" curve cannot be even roughly estimated, as stressed in my earlier paper (SEKANINA 1960), and the fact cannot be eliminated that the former factors yield the changes in order greater than the latter factors. Fig. 8 gives the long-term course of the characteristics $N_y^{(o)}$, $\tau_y^{(o)}$ and $m^{(o)}$ as observed in the West and Central European area.

The curves $N_y^{(o)}$, and $\tau_y^{(o)}$ indicate a sharp maximum round 1855, when about 2.4 times as many comets were discovered as in the following minimum, falling on 1890. In all the probability, the comet hunters BRUHNS, DE VICO, BRORSEN, KLINKERFUES and others are responsible for the sharp maximum at the middle of the last century.

The development of observational technique is well seen on the apparent-brightness curve, which appears, on the general, only small deviations from the straight line. In spite of this, in the region of higher values of $N_y^{(o)}$ and $\tau_y^{(o)}$ a more rapid decrease of the brightness curve $m^{(o)}$ is apparent, proving good instrumental equipments of the comet hunters. If the form of the curves of the "comet meteorology" quantities were influenced by the fluctuations in the number of comets, the course of the $m^{(o)}$-curve would have to follow the curves $N_y^{(o)}$ and $\tau_y^{(o)}$ (cf. what has been said about the relations among the individual members of the "comet meteorology" quantities at the end of the foregoing section).

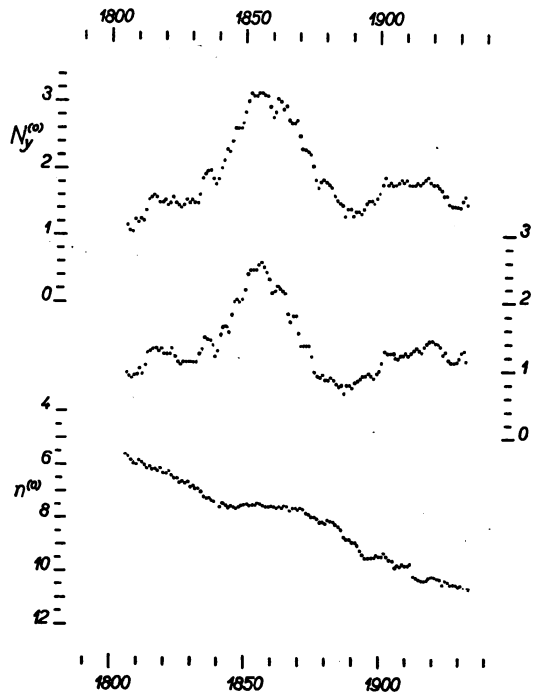

Fig. 8. The long-term variations of the "comet meteorology" quantities in West and Central Europe.

It is important to compare the course of the quantities of Fig. 8 with the form of the curve of cometary discoveries visible with the naked eye (Section 14 of my former paper [SEKANINA 1960]), which shows that their mutual relation is often opposite, i. e., for instance, at the time, when the total number of cometary discoveries is increasing, the number of discoveries of bright comets is dropping. Concluding, the set of the comets visible with the naked eye studied in my former paper is for the purpose of the indication of long-term variations in night-cloudiness much more suitable than the sets of $N_y^{(o)}, \tau_y^{(o)}$ or $m^{(o)}$ owing to the fact that the subjective factors are in the former set reduced to their minimum.

6. SHORT-TERM AND LONG-TERM VARIATIONS OF THE "COMET METEOROLOGY" QUANTITIES IN THE UNITED STATES

The values of the quantities of "cometary meteorology" smoothed out of three neighbouring yearly data, $N_y(t)$, $\tau_y(t)$, $m(t)$, and freed of the long-term variations, $N_y^{(o)}$, $\tau_y^{(o)}$, $m^{(o)}$, are again denoted by the symbols ΔN_y, $\Delta\tau_y$ and Δm, and their course is represented in Fig. 9. The long-term-variation curves $N_y^{(o)}$, $\tau_y^{(o)}$, $m^{(o)}$ are included in Fig. 10. The parameters of the curves ΔN_y, $\Delta\tau_y$, Δm are listed in Table 8, from which a good agreement results both between their period variations and between the average positions of the extremes. Table 9 contains the correlation coefficient $\psi^*(o, e)$ for each quantity. The long-term courses of the curve of cometary discoveries and of the tail function as compared with the same curves for West and Central Europe show no mutual agreement. A conspicuous maximum falls here on the period round 1890, i. e. in the same time, when a

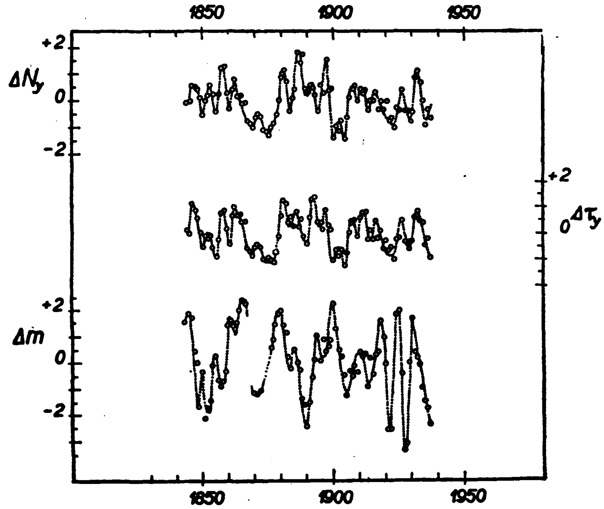

Fig. 9. The short-term variations of the "comet meteorology" quantities in the United States.

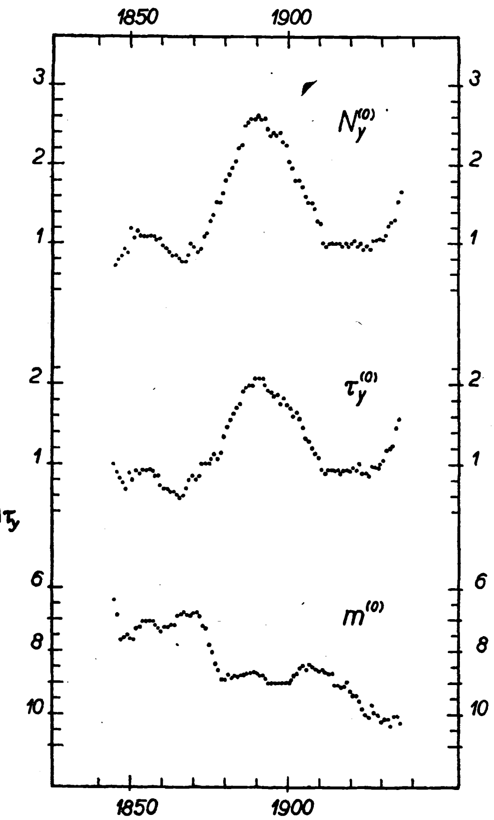

Fig. 10. The long-term variations of the "comet meteorology" quantities in the United States.

Table 8.

Time parameters of the curves of the "comet meteorology" quantities in the United States

	ΔN_y	$\Delta \tau_y$	Δm
P_0	6.11 ± 0.24	6.07 ± 0.23	6.35 ± 0.19
T_0 (min)	1905.25 ± 0.32	1905.58 ± 0.36	1904.82 ± 0.54
T_0 (max)	1902.12 ± 0.40	1902.11 ± 0.39	1901.72 ± 0.58
φ_0	0.49 ± 0.06	0.43 ± 0.06	0.51 ± 0.05

Table 9.

The correlation degree (o, e) of the "comet meteorology" quantities in the United States

quantity	$\psi^* (o, e)$
ΔN_y	+0.12
$\Delta \tau_y$	—0.27
Δm	+0.23

minimum takes place in the European Continent. It proves once more that the comet hunters have the main influence on the comet discoveries. The end of the 19th century is characterized by the activity of a number of American comet hunters, such as BROOKS, BARNARD, SWIFT, PERRINE etc. Their instrumental equipment affect the form of the $m^{(o)}$-curve at the bottom of Fig. 10.

7. CONCLUSIONS

Analyzing the curves of the quantities of "comet meteorology", i. e. of the cometary discoveries, N_y, of the function of cometary tails, τ_y, and of the apparent brightness, m, of comets at the time of discovery, and their connections with the precipitation activity I have arrived at the conclusions as follows:

(1) The curves of the "comet meteorology" quantities, being free of the long-term variations, appear in the West and Central European area the variations of a period of 6.3 ± 0.2 years, with the basic maximum at $1902.9 \pm \pm 0.5$ and the basic minimum at 1905.9 ± 0.4. The period length is varying in the limits of 4 to 12 years.

(2) The curves of the quantities of "cometary meteorology" in the West and Central European area indicate period variations, the periodicity of which is about 60 years for the apparent-brightness curve, and more than 100 years for the curves of comet discoveries and of the tail function, a number of probably unreal fluctuations being superimposed on the two latter mentioned.

(3) The connection between the precipitation activity and the quantities of "cometary meteorology" in West and Central Europe is studied in two stages. Firstly, the relation is investigated regardless of the existence of period variations, secondly including the latter. The conclusion is arrived at that the correlations between the precipitation and cometary discoveries, or the tail function, are real, while the connection between the precipitation and the apparent brightness is less conspicuous. These relations must be understood: the greater precipitation activity the less comets are discovered and the comets are less bright in the sky. It is proved that in most cases the decrease of the coefficients of these correlations is due to the mutual phase shift of the curves. Possible effects of both accidental and systematic (physical) natures are pronounced

of the shift of the precipitation activity relative to the curves of the "comet meteorology" quantities.

(4) The mutual connections among the "comet meteorology" quantities in the West and Central European area is investigated in two stages, too. The results obtained in quite different ways in shorter intervals of time in my former paper (SEKANINA 1960) are confirmed: the correlation degree of comet discoveries (or of the tail function) with the apparent-brightness curve is comparatively very low. In the course of a longer period, however, the two curves comply with one another better and better, which results, for instance, from the agreement of the basic moments of the extremes on the two curves. The agreement between the curve of comet discoveries and that of the tail function is very good.

(5) The curves of the "comet meteorology" quantities in the United States appear the properties similar to those in West and Central Europe, as described at item (1). The period of the variations is 6.2 ± 0.2 years on the average, the basic maximum is at 1902.0 ± 0.5 and the basic minimum at 1905.2 ± 0.4. The individual periods have the limits of 3 to 10 years.

(6) The long-term variations in the "comet meteorology" quantities in West and Central Europe differs considerably from those in the United States. In Europe a conspicuous maximum of comet discoveries occurred at the middle of the 19th century, while in the United States at its end. At the same time the increase of comet discoveries (and of the tail function) is followed (particularly in the U. S.) by a decrease on the apparent-brightness curve. This effect corroborates the opinion that the long-term variations as found from the observational material are of no physical nature, but that they are produced by the activity of comet hunters. In Europe as well as in the U. S. the period of activity of the famous comet hunters is identical with the period of maximum comet discoveries. If the long-term variations of the three characteristics were of physical nature, they would have to comply with one another, in accordance with what has been said at item (4). In practice, however, the empirical form of the apparent-brightness curve is quite opposite to those of comet discoveries and of the tail function. The instrumental equipment of the comet hunters makes the main effect here.

In the conclusion, my thanks belong to Dr. L. KŘIVSKÝ, CSc., Astronomical Institute of the Czechoslovak Academy of Sciences, Ondřejov, for a careful scanning the manuscript and for a number of useful notes made in the discussions on the problem.

REFERENCES

BOUŠKA J., 1950, Bull. Mét. Tchecoslovaquie 4, 76.
HELLMANN G., 1909, Veröf. Preuss. Met. Inst. Nr. 207.
LINK F., VANÝSEK V., 1947, CR 225, 1284.
SEKANINA Z., 1959, BAC 10, 103.
SEKANINA Z., 1960, BAC 11, 94.
SEKANINA Z., 1964, Publ. Astron. Inst. Charles Univ. Prague, Ser. II No 42.

ATMOSFÉRICKÉ SRÁŽKY A PŘÍMO MĚŘENÉ CHARAKTERISTIKY KOMET

Je zkoumán krátkodobý i dlouhodobý chod tří kometárních charakteristik bezprostředně pozorovaných: kometárních objevů, funkce mohutnosti kometárních chvostů a střední zdánlivé jasnosti komet v době jejich objevu, jež jsou v práci souhrnně nazývány veličinami „kometární meteorologie", v souvislosti s chodem srážek, a to podrobně v oblasti západní a střední Evropy a v hlavních rysech i v oblasti U.S.A. Je zjištěno, že kolísání veličin „kometární meteorologie" i srážek probíhá ve stejné periodě asi 6,2 až 6,3 roku, a to v obou vyšetřovaných oblastech. Vzájemné srovnání průběhu jednotlivých veličin „kometární meteorologie" v západní a střední Evropě dokazuje jednak, že korelace mezi prvními dvěma z nich je velmi dobrá a jednak, že zdánlivá jasnost nejeví korelaci s oběma prvně jmenovanými veličinami v kratších časových intervalech, ale že tato korelace existuje u vyrovnaných hodnot v delších intervalech. Porovnání chodu srážek s křivkami veličin „kometární meteorologie" v západní a střední Evropě pak vede k závěru, že souvislost mezi nimi je reálná a lze ji přibližně formulovat takto: v době maximálních srážek dosahují veličiny „kometární meteorologie" svých minimálních hodnot a naopak. Tato korelace zároveň potvrzuje i oprávněnost termínu, veličiny „kometární meteorologie"", pro uvedené kometární charakteristiky. Dále se dokazuje, že dlouhodobý chod veličin „kometární meteorologie" v obou oblastech je v podstatě obrazem činnosti lovců komet a nelze ho proto porovnávat s dlouhodobým chodem srážek.

АТМОСФЕРНЫЕ ОСАДКИ И ПРЯМО ИЗМЕРЕННЫЕ ХАРАКТЕРИСТИКИ КОМЕТ

Резюме

В настоящй работе определяется период колебаний основных кометных характеристик первого класса, называемых здесь величинами „кометной метеорологии" вследствие их физического значения, и исследуются отношения между этими характеристиками и атмосферными осадками в западной и центральной Ервопе и в Соединенных Штатах Америки. Средний период колебаний кометных характеристик 6,3 лет в Европе и 6,2 лет в США. Корреляция между кометными открытиями и осадками достаточно высока и показывает, что чем более осадков, тем менее обнаруженных комет. На вековые колебания характеристик повлияют прежде всего ловцы комет. Все зависимости чисто статистического характера.