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ANALYSIS OF STRUCTURAL PROPERTIES OF PETRI NETS BASED ON PRODUCT INCIDENCE MATRIX

Guangyou Ji and Mingzhe Wang

This paper presents some structural properties of a generalized Petri net (PN) with an algorithm to determine the (partial) conservativeness and (partial) consistency of the net. A product incidence matrix $A = CC^T$ or $\tilde{A} = C^T C$ is defined and used to further improve the relations among PNs, linear inequalities and matrix analysis. Thus, based on Cramer's Rule, a new approach for the study of the solution of a linear system is given in terms of certain sub-determinants of the coefficient matrix and an efficient algorithm is proposed to compute these sub-determinants. The paper extends the common necessary and/or sufficient conditions for conservativeness and consistency in previous papers and some examples are designed to explain the conclusions finally.

Keywords: Petri net, structural property, linear inequality, product incidence matrix

Classification: 93C65, 93A15

1. INTRODUCTION

A Petri net (PN) is a mathematical model which aims to analyze the performance of manufacturing systems, communication networks, business management, web services, and so on (see [10, 12, 13] for further details). The analysis of a PN model aims to investigate its structural and behavioral properties. But with the increase of the scale of the systems, some computational difficulties would arise, which may prevent us from getting a clear view of the influence of various factors on the dynamic behavior of the systems. Many researchers are thus motivated to investigate better methods and techniques for analyzing structural properties such as boundness, (partial) conservativeness, (partial) repetitiveness and (partial) consistency (see [7, 10]). The reachability graph or coverability graph method was proposed in [8], which followed the order of the PN model, and required the enumeration of all the reachable markings of the net. Unfortunately, this method suffered from the state space explosion problem. Thus, the reduction technique, which can preserve certain structural properties of the PNs, was introduced to analyze some large PN models. However, if these PNs can not represent the required properties, it would be very difficult to find and correct the design mistakes with this method (see [2, 3, 21]). A more efficient method to explore structural properties of PN models is the algebraic method, which can analyze the properties directly based on the incidence matrices of PNs without self-loop. The algebraic method based on the incidence matrix for structural analysis presents advantages over other approaches such as the reachability graph or the reduction method. First, it can avoid the state space explosion problem and be carried out on a computer. Second, some useful information about the dynamic behavior can be obtained by using structural analysis (see [1, 5, 17, 19]).

Up to the present, there is no one common analytical method for a general PN, whereas the linear-algebra-based method for structural analysis is remarkably successful in analyzing some subclasses of pure PNs such as choice-free PNs, forward (or backward)-conflict-free, and forward (or backward)-concurrent-free PNs (see [18, 19]). In [14], the authors have further perfected the linear-algebra method for the structural analysis of general PNs. Some structural properties of general PNs have been shown, which improves the link between PNs and the linear-algebra techniques. Firstly, some conditions are given to detect the structural boundedness and repetitiveness of PNs by eigenvalues analysis of their incidence matrices, which must be a square matrix. Unfortunately, the incidence matrices of most PNs are not square, and thus the eigenvalues of their incidence matrices do not exist. To overcome the limitation, we consider the product incidence matrix $A = CC^T$ or $\tilde{A} = C^TC$, where C is the incidence matrix of a PN. By investigating the product incidence matrix, the relations are further improved among PNs, linear inequalities and matrix analysis. Secondly, the algebraic tests are given which are suitable for digital computation for property analysis of PNs. The tests have to be carried out by using either elementary scalar products or sub-determinants of the incidence matrix. Based on the Cramer′s Rule (see [6]), we will continue to investigate the structural properties of general PNs by computing determinants of special sub-matrices of the incidence matrix. Some examples are given to explain our conclusions.

In this paper, new algebraic methods based on product incidence matrices for structural analysis of PN models are proposed. This is rarely found in previous papers of PN theory. Besides, a new method is proposed for computing P(T)-invariants and checking the conservativeness, consistency of PNs based on special sub-determinants of the incidence matrix of the net. So far, the Fourier-Motzkin (FM) algorithm is the most generally used method for computing invariants, the biggest disadvantage of which is that the number of invariants in a given net may grow exponentially (see [7, 15, 16, 20]). Our methods are implemented on a computer by MATLAB and LINDO, which makes the structural analysis of general PN models easier.

The rest of the paper is organized as follows. Some basic concepts and notations about matrix theory and PN are introduced in Section 2. Some propositions are obtained and proved about conservativeness, consistency of a general PN by analyzing the product incidence matrices in Section 3. Linear inequalities are generated by the linear combinations of some sub-determinants of the incidence matrix and the properties of general PNs are tested by the alternating signs of these linear inequalities in Section 4. Several examples are given to illustrate the results in Section 5. Finally, Section 6 concludes this paper.

2. PRELIMINARIES AND BASIC DEFINITIONS

2.1. Matrix

In this paper, m and n are fixed positive integers (unless otherwise specified). The set of nonnegative integers and real numbers is denoted by N and R, respectively. Given matrices $A = (a_{ij})$ and $B = (b_{ij})$ in $R^{n\times n}$. A is said to be nonnegative (or positive), denoted as $A \geq 0$ (or $A > 0$), if $a_{ij} \geq 0$ (or $a_{ij} > 0$), $\forall i, j \in H_n = \{1, 2, \ldots, n\}$. Define $A \geq B$ for $A - B \geq 0$, if $a_{ij} \geq b_{ij}$, for $\forall i, j \in H_n$. Also, a column vector $x = (x_i)^T$ in R^n is said to be nonnegative (or positive), denoted as $x \geq 0$ (or $x > 0$), if $x_i \geq 0$(or $x > 0$), for $\forall i \in H_n$. A^T is the transpose of a matrix A. The determinant of a matrix A will be denoted by $det(A)$. A matrix $A = (a_{ij})$ is said to be reducible if there exists a non-void set $F \subset H_n$, such that $a_{ij} = 0$ for $i \in F$ and $j \in H_n - F$. A matrix A is irreducible if it is not reducible (see [4]).

Definition 2.1. (see Hefferon [6])

(1) The number $det(A_{ij})$ is called the (i, j)th minor of A, where A_{ij} is the matrix of order $n - 1$ obtained by deleting the ith row and jth column of A.

(2) The (i, j) cofactor of A, denoted M_{ij}, is the number $(-1)^{i+j}det(A_{ij})$. The adjugate matrix of A is defined by $adj(A) = (M_{ij})^T$.

Definition 2.2. (see Fiedler and Pták [4]) We shall denote by Z the class of all real square matrices whose off-diagonal elements are all non-positive (i. e. A matrix A is said to be in Z if and only if all off-diagonal entries of A are non-positive, for a given matrix $A \in R^{n\times n}$).

Proposition 2.3. (see Fiedler and Pták [4]) If $A \in Z$ is irreducible and singular, and all real eigenvalues of A are nonnegative, then there exists a vector $x > 0$ such that $Ax = 0$.

Fact 2.4. (see Hefferon [6]) Given any real matrix $C = (c_{ij})_{m\times n}$, then the following facts are salient:

(1) $ker(C^T) = ker(CC^T) = \{x \in R^m \mid C^T x = 0\}$;

(2) $ker(C) = ker(C^T C) = \{x \in R^n \mid Cx = 0\}$.

Note that the two facts explain the solution spaces of the equations $CC^T x = 0$ and $C^T Cx = 0$ is equivalent to the solution spaces of the equations $C^T x = 0$ and $Cx = 0$, respectively.

2.2. Petri nets

In this section, we recall the definitions and properties of a general PN (see [10, 11]). A PN structure is defined by a 4-tuple $\Sigma = (P, T, Pre, Post)$, where $P = \{p_1, p_2, \ldots, p_m\}$ is a finite set of places represented by circles, $T = \{t_1, t_2, \ldots, t_n\}$ is a finite set of transitions represented by bars; $Pre : P \times T \to N$ is the input functions identifying the relation from the places to the transitions; $Post : P \times T \to N$ is the output functions identifying the relation from the transitions to the places. The incidence matrix C of a PN is an

$m \times n$ matrix defined as $C = Post - Pre$, where $C(i,j) = Post(p_i, t_j) - Pre(p_i, t_j)$ describes the change in the number of tokens in place p_i when transition t_j fires. In the whole paper, it is assumed that a PN is pure, i.e. it has no self-loop, which satisfies $Pre(p,t)Post(p,t) = 0$, $\forall p \in P$ and $\forall t \in T$. A PN Σ is k-bounded if the number of tokens in each place does not exceed a finite number k for any reachable marking. A PN Σ is structurally bounded if there exists an m-vector $x \in N^m$, $x > 0$ such that $C^T x \leq 0$. A PN Σ is said to be (partially) conservative if there exists an m-vector $x \in N^m$, $x > 0$ ($x \geq\neq 0$) such that $C^T x = 0$. A vector $x \geq\neq 0$ is called P-invariant, if it satisfies $C^T x = 0$.

A PN Σ is (partially) repetitive if there exists an n-vector $y \in N^n$, $y > 0$ ($y \geq\neq 0$) such that $Cy \geq 0$. A PN Σ is said to be (partially) consistent if there exists an n-vector $y \in N^n$, $y > 0$ ($y \geq\neq 0$) such that $Cy = 0$. A vector $y \geq\neq 0$ is called T-invariant, if it satisfies $Cy = 0$ (see [10]).

3. RESULTS ON THE PRODUCT INCIDENCE MATRIX

In [10], necessary and/or sufficient conditions are provided for some important structural properties of a PN, such as structural boundedness, conservativeness, repetitiveness and consistency. They only depend on the topological structures of PNs and are independent of the initial marking. Therefore, the structural properties of a pure PN can be completely described by the incidence matrix C and its associated equations or inequalities. There have been many well-known methods based on incidence matrix for structural properties (see [7]). In this section, some structural properties will be derived by computing a product incidence matrix $A = CC^T = (a_{ij}) \in R^{m \times m}$ (i.e. it represents the relation between places) or $\tilde{A} = C^T C = (\tilde{a}_{ij}) \in R^{n \times n}$ (i.e. it represents the relation between transitions), where $C_{m \times n}$ is the incidence matrix of the Petri net Σ. Some results are presented on the conservativeness and consistency by using the product incidence matrix. According to [9], a PN Σ is well structured if it is both conservative and consistent. We will give, in the following, a characterization for a well-structured PN.

Proposition 3.1. Let $C \in R^{m \times n}$ be the incidence matrix of a PN Σ. If the following conditions hold:

(1) The product incidence matrix $A = CC^T$ is singular;

(2) In the adjugate matrix $adj(A) = (M_{ij})^T$, $i, j \in H_m$, there is at least one column or row, whose elements are either all positive or all negative (i.e. there exists at least one j, such that $M_{ij} > 0$ or $M_{ij} < 0$, for $\forall i \in H_m$).

Then the PN Σ is conservative.

Proof. According to Laplace′s expansion (see [6]) for the determinant of a square matrix $A \in R^{m \times m}$, we have $A \cdot adj(A) = adj(A) \cdot A = det(A) \cdot E_m$, where E_m is an $m \times m$ identity matrix. Since the matrix A is singular, then $det(A) = 0$. So $A \cdot adj(A) = 0$, i.e. $A \cdot (M_{ij})^T = 0$. Thus, each column of the adjugate matrix $adj(A)$ is denoted as $x_i = (M_{i1}, M_{i2}, \ldots, M_{im})^T$, $i \in H_m$, then we have $Ax_i = 0$, $i \in H_m$. Without loss of generality, we suppose every element of the ith column are either positive or negative, i.e. for $i \in H_m$, $M_{ij} > 0$ or $M_{ij} < 0$, $\forall j \in H_m$. There are two cases to be considered:

Case 1): For some $i \in H_m$, $M_{ij} > 0$, $\forall j \in H_m$, there exists a positive vector $u = x_i > 0$ such that $Au = 0$, that is $CC^T u = 0$. By Fact 2.4(1), we have $C^T u = 0$. Therefore, the PN Σ is conservative.

Case 2): For some $i \in H_m$, $M_{ij} < 0$, $\forall j \in H_m$, there exists a positive vector $u = -x_i > 0$ such that $Au = 0$, that is $CC^T u = 0$. By Fact 2.4(1), we have $C^T u = 0$. Therefore, the PN Σ is conservative. □

Remark 3.2.

- The singularity assumption about A is required, since otherwise $Ax = 0$ would have only the trivial solution $x = 0$ (see Proposition 3.10 and Proposition 3.11).
- In Proposition 3.1, if 3.1(2) is replaced with 3.1(2′): In the adjugate matrix $adj(A) = (M_{ij})^T$, there is at least one column or row, whose elements are either all non-positive or all non-negative. Then the PN Σ is partially conservative.

Proposition 3.3. Let $C \in R^{m\times n}$ be the incidence matrix of a PN Σ. If the following conditions hold:

(1) The product incidence matrix $\tilde{A} = C^T C$ is singular;

(2) In the adjugate matrix $adj(\tilde{A}) = (\tilde{M}_{ij})^T$, $i, j \in H_n$, there is at least one column or row, whose elements are either all positive or all negative (i. e. there exists at least one j, such that $\tilde{M}_{ij} > 0$ or $\tilde{M}_{ij} < 0$, for $\forall i \in H_n$).

Then the PN Σ is consistent.

Proof. This is obtained by the above method and by Fact 2.4(2). □

Remark 3.4.

- In Proposition 3.3, if 3.3(2) is replaced with 3.3(2′): In the adjugate matrix $adj(\tilde{A}) = (\tilde{M}_{ij})^T$, there is at least one column or row, whose elements are either all non-positive or all non-negative. Then the PN Σ is partially consistent.
- Furthermore, it is noted that the above propositions hold in the case of the rank of the incidence matrix C is $m-1$ or $n-1$. According to the definition of an adjugate matrix, it is well known that the adjugate matrix of the product incidence matrix $A = CC^T$ or $\tilde{A} = C^T C$ is zero matrix, when the rank of incidence matrix C is less than $m-1$ or $n-1$.
- Since the product incidence matrices $A = CC^T$ and $\tilde{A} = C^T C$ are symmetric, their adjugate matrices are also symmetric. Thus the complexity of the cofactors of $A = CC^T$ or $\tilde{A} = C^T C$ to be computed is less than $m(m+1)/2$ or $n(n+1)/2$, respectively. Hence, comparing to that of [14], the computational complexity of these results is reduced.

Based on the above conclusions, we obtain the following important theorem.

Theorem 3.5. Let C be the incidence matrix of a PN Σ. The PN is well structured, if the following conditions hold:

(1) If the two matrices $A = CC^T$ and $\tilde{A} = C^T C$ are singular;

(2) In the adjugate matrices $adj(A)$ and $adj(\tilde{A})$, both have at least one column or row, whose elements are either all positive or all negative.

Proof. Since both Proposition 3.1 and Proposition 3.3 are satisfied, the conclusion holds. □

Example 3.6. (see Murata [10]) The PN Σ shown in Fig.1 represents deterministic parallel activities. Activities are represented by t_2 and t_3. They begin after the firing of transition t_1 and end after the firing of transition t_4. Its incidence matrix C is

$$C = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 1 \end{pmatrix}.$$

Then, we have $\tilde{A} = C^T C = \begin{pmatrix} 3 & -1 & -1 & -1 \\ -1 & 2 & 0 & -1 \\ -1 & 0 & 2 & -1 \\ -1 & -1 & -1 & 3 \end{pmatrix}$,

$$adj(\tilde{A}) = adj(C^T C) = \begin{pmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{pmatrix}.$$

Hence, the first column of $adj(\tilde{A})$ is

$$x_1 = \begin{pmatrix} M_{11}, M_{12}, M_{13}, M_{14} \end{pmatrix}^T = \begin{pmatrix} 8, 8, 8, 8 \end{pmatrix}^T > 0.$$

From Proposition 3.3, the PN Σ is consistent.

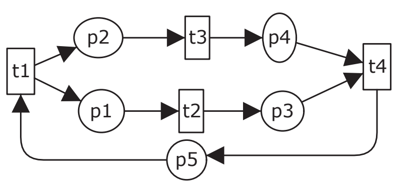

Fig. 1. A PN representing parallel activities

We will show the eigenvalue method for analyzing conservativeness and consistency of a PN based on the following fact.

Fact 3.7. (see Murata [10]) If $C \in R^{m\times n}$ is any real non-zero matrix, then all the eigenvalues of the symmetric matrices $A = CC^T$ and $\tilde{A} = C^T C$ are nonnegative real numbers.

Proposition 3.8. If $A = CC^T \in Z$ is irreducible and singular, then the PN Σ is conservative.

Proof. According to Fact 3.7 and Proposition 2.3, there exists $x > 0$ such that $Ax = CC^T x = 0$. By Fact 2.4(1), we have $C^T x = 0$. Hence, the PN Σ is conservative. □

Proposition 3.9. If $\tilde{A} = C^T C \in Z$ is irreducible and singular, then the PN Σ is consistent.

Proof. According to Fact 3.7 and Proposition 2.3, there exists $x > 0$ such that $\tilde{A}x = C^T Cx = 0$. By Fact 2.4(2), we have $Cx = 0$. Hence, the PN Σ is consistent. □

Proposition 3.10. If $A = CC^T \in Z$ and $\tilde{A} = C^T C \in Z$ is irreducible and singular, then the PN Σ is well structured.

Proof. Because the conditions of Proposition 3.6 and Proposition 3.7 are satisfied, the conclusion holds. □

Example 3.11. Let a PN Σ be as shown in Figure 2. Its product incidence matrix $C^T C$ is

$$C^T C = \begin{pmatrix} 2 & 0 & 0 & -1 & 0 & -1 & 0 & 0 \\ 0 & 3 & 0 & -1 & 0 & 0 & -1 & -1 \\ 0 & 0 & 4 & 0 & -1 & 0 & -1 & -2 \\ -1 & -1 & 0 & 4 & 0 & -1 & -1 & 0 \\ 0 & 0 & -1 & 0 & 2 & 0 & 0 & -1 \\ -1 & 0 & 0 & -1 & 0 & 2 & 0 & 0 \\ 0 & -1 & -1 & -1 & 0 & 0 & 3 & 0 \\ 0 & -1 & -2 & 0 & -1 & 0 & 0 & 4 \end{pmatrix}.$$

It is easy to know that $\tilde{A} = C^T C \in Z$ is irreducible and singular, then the PN Σ is consistent by Proposition 3.9.

Proposition 3.12. If $A = CC^T$ is nonsingular, then PN Σ is not conservative.

Proof. If $A = CC^T$ is nonsingular, no eigenvalue of A can be zero. The only solution of equation $Ay = CC^T y = 0$ is $y = 0$, that is, the only solution of equation $C^T y = 0$ is $y = 0$ by Fact 2.4(1). The PN Σ is not conservative. □

Proposition 3.13. If $\tilde{A} = C^T C$ is nonsingular, then PN Σ is not consistent.

Proof. The proof is similar to that of Proposition 3.12. □

It means that non-conservation and non-consistency of a PN can be determined by the rank of its incidence matrix $C_{m\times n}$, that is, $rank(C) = m$, then PN is not conservative, or $rank(C) = n$, then PN is not consistent.

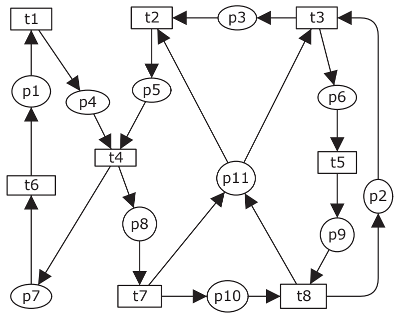

Fig. 2. An augmented marked graph

4. ANALYSIS OF STRUCTURAL PROPERTIES

Rachid and Abdellah adopted determinants of special sub-matrices of the incidence matrix to describe necessary and/or sufficient conditions for structural boundedness, repetitiveness, conservativeness, and consistency for some general PNs. However, the rank of the incidence matrices of these PNs must be equal to $m-1$ or $n-1$ (see [14]). Thus, if the required conditions are not satisfied, it is hard to test sub-determinants alternating in sign when the scale of the system increases. In this section, based on the methods of [14] and the Cramer′s Rule (see [6]), we will continue to analyze structural properties of general PNs without self-loops.

Given a matrix $C_{m\times n}$ whose rank is r, denoted as $rank(C_{m\times n}) = r$, then there exists r rows which are linearly independent. Without loss of generality, suppose that the first r rows of C are linearly independent (it is always possible to permute rows of C such that this condition is met). Denoted as

$$\hat{C} = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1n} \\ c_{21} & c_{22} & \dots & c_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ c_{r1} & c_{r2} & \dots & c_{rn} \end{pmatrix}. \tag{1}$$

Let $u = n - r$, denote $C_{j+1,j+2,\dots,j+u}$ as a square matrix of order $n-u=r$ obtained from $\hat{C}$ by deleting its $j+1, j+2, \dots, j+u$th columns, where $j \in \{0, 1, \dots, r\}$. According to the knowledge of linear algebra and Cramer′s Rule (see [6]), we have the following Lemma and Corollary.

Lemma 4.1. Given a linear system $Cx = 0$ and $rank(C_{m\times n}) = r$, then the solution space $S = \{x \in R^n \mid Cx = 0\}$ can be expressed by

$$S = \{x = (x_1, x_2, \dots, x_n)^n \mid x_i = (-1)^{i-1}[\rho_1 det(C_{i,j+2,j+3,\dots,j+u}) + \rho_2 det(C_{i,j+1,j+3,\dots,j+u}) + \dots + \rho_u det(C_{i,j+1,j+2,\dots,j+u-1})],\ \forall i \in H_n\} \tag{2}$$

where $\rho_1, \rho_2, \dots, \rho_u$ are proper constants and independent of i, and $n-u=r$.

P r o o f. It can be easily proved by the Cramer′s Rule and the algebra technique. □

Note that:

(i) $det(C_{j_1,\ldots,j_i,\ldots,j_k,\ldots,j_u})=0$, if $j_i = j_k$, for some $j_i, j_k \in \{j+1, j+2, \ldots, j+u\}$, $i \neq k$. Otherwise $det(C_{j_1,\ldots,j_i,\ldots,j_k,\ldots,j_u}) = (-1)^{\tau(j_1 j_2 \ldots j_u)} det(C_{j+1,j+2,\ldots,j+u})$, if $j_i \neq j_k$, for $\forall i, k \in \{1, 2, \ldots, u\}$, $i \neq k$, where $\tau(j_1 j_2 \ldots j_u)$ is the reversal number of the sequence $j_1, j_2, \ldots, j_u$.

(ii) Owing to $rank(C_{m\times n}) = r = n - u$, there exists at least one $det(C_{j+1,j+2,\ldots,j+u})$ such that $det(C_{j+1,j+2,\ldots,j+u}) \neq 0$. A matrix $D_{n\times u}$ denoted as

$$\begin{pmatrix} det(C_{1,j+2,\ldots,j+u}) & det(C_{1,j+1,\ldots,j+u}) & \ldots & det(C_{1,j+1,\ldots,j+u-1}) \\ \vdots & \vdots & \vdots & \vdots \\ det(C_{i,j+2,\ldots,j+u}) & det(C_{i,j+1,\ldots,j+u}) & \ldots & det(C_{i,j+,\ldots,j+u-1}) \\ \vdots & \vdots & \vdots & \vdots \\ det(C_{n,j+2,\ldots,j+u}) & det(C_{n,j+1,\ldots,j+u}) & \ldots & det(C_{n,j+1,\ldots,j+u-1}) \end{pmatrix}.$$

Thus $x = diag(1, -1, \ldots, (-1)^{i-1}, \ldots, (-1)^{n-1}) \cdot D_{n\times u} \cdot (\rho_1, \rho_2, \ldots, \rho_u)^T$ is a general solution of the linear system $Cx = 0$.

Corollary 4.2. Given a linear system $Cx = 0$ and $rank(C_{m\times n}) = r = n - 2$ (i. e. $u = 2$), then $S = \{x \in R^n \mid Cx = 0\}$ can be expressed by

$$S = \{x = (x_1, x_2, \ldots, x_n)^n \mid x_i = (-1)^{i-1}[\rho_1 det(C_{i,k}) + \rho_2 det(C_{i,l})],\ \forall i \in H_n, k < l\}$$

where ρ_1 and ρ_2 are proper constants and independent of i.

Example 4.3. Let us consider the linear system:

$$\begin{cases} x_1 - x_2 + x_3 - 2x_5 = 0 \\ x_2 - x_3 + x_4 - 3x_5 = 0 \\ \quad -x_1 - x_4 + 5x_5 = 0. \end{cases}$$

Its coefficient matrix is

$$C = \begin{pmatrix} 1 & -1 & 1 & 0 & -2 \\ 0 & 1 & -1 & 1 & -3 \\ -1 & 0 & 0 & -1 & 5 \end{pmatrix},$$

and $rank(C) = 2 = 5 - 3$, then we have $\hat{C} = \begin{pmatrix} 1 & -1 & 1 & 0 & -2 \\ 0 & 1 & -1 & 1 & -3 \end{pmatrix}$.

Observing the matrix $\hat{C}$, we get $det(C_{345}) = \begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} = 1 \neq 0$, i. e. C_{345} is a 2×2 matrix obtained from $\hat{C}$ by deleting its 3,4,5th columns, so we can choose $j = 2$. Thus, any solution of the linear system can be expressed as:

$$\begin{cases} x_1 = \rho_1 det(C_{145}) + \rho_2 det(C_{135}) + \rho_3 det(C_{134}) \\ x_2 = -[\rho_1 det(C_{245}) + \rho_2 det(C_{235}) + \rho_3 det(C_{234})] \\ x_3 = \rho_1 det(C_{345}) + \rho_2 det(C_{335}) + \rho_3 det(C_{334}) \\ x_4 = -[\rho_1 det(C_{445}) + \rho_2 det(C_{435}) + \rho_3 det(C_{434})] \\ x_5 = \rho_1 det(C_{545}) + \rho_2 det(C_{535}) + \rho_3 det(C_{534}). \end{cases}$$

So, we have $D_{5\times 3} = \begin{pmatrix} 0 & -1 & 5 \\ -1 & 1 & -3 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

Thus, $x = diag(1, -1, 1, -1, 1) \cdot D_{5\times 3} \cdot (\rho_1, \rho_2, \rho_3)^T$ is a general solution of the linear system.

By the Appendix′s algorithm, we can obtain a positive solution $x = (4, 3, 1, 1, 1)^T$ by choosing the constants $\rho_1 = \rho_2 = \rho_3 = 1$ (see Example 5.1.).

Some interesting results arise from the above Lemma and Corollary for conservativeness and consistency of PNs which will be shown next.

Theorem 4.4. Let r be the rank of the incidence matrix $C_{m\times n}$ of a PN, and let $\hat{C}$ and $C_{j+1,j+2,\ldots,j+u}$, $j \in \{0, 1, \ldots, r\}$ be as defined above. If $r = m - u$, $1 \le u < m$, then the PN is conservative if and only if there exist non-zero constants $\rho_1, \rho_2, \ldots, \rho_u$ such that

$$\begin{aligned} &[\rho_1 det(C_{i,j+2,\ldots,j+u}) + \rho_2 det(C_{i,j+1,\ldots,j+u}) + \ldots + \rho_u det(C_{i,j+1,\ldots,j+u-1})] \\ &\times[\rho_1 det(C_{i+1,j+2,\ldots,j+u}) + \rho_2 det(C_{i+1,j+1,\ldots,j+u}) + \ldots + \rho_u det(C_{i+1,j+1,\ldots,j+u-1})] < 0, \\ &\forall i \in H_{m-1}. \end{aligned} \tag{3}$$

Proof. Necessity: If the PN is conservative, then according to the definition of the conservativeness, there exists an integer vector $x > 0$ of order m such that $C^T x = 0$. This implies:

$$\sum_{i=1}^{m} c_{ij} x_i = 0, \ \forall j \in H_n. \tag{4}$$

Since the rank of C is $r = m - u$, by Lemma 4.1, the solution of these equations takes the form of (2),

$$\begin{aligned} x_i = (-1)^{i-1}\{&\rho_1 det(C_{i,j+2,j+3,\ldots,j+u}) + \rho_2 det(C_{i,j+1,j+3,\ldots,j+u}) + \ldots \\ &\ldots + \rho_u det(C_{i,j+1,j+2,\ldots,j+u-1})\}. \end{aligned}$$

Since $x_i > 0$ for $\forall i \in H_m$, we have the inequality $x_i x_{i+1} > 0$. It implies

$$\begin{aligned} &(-1)^{2i-1}[\rho_1 det(C_{i,j+2,\ldots,j+u}) + \rho_2 det(C_{i,j+1,\ldots,j+u}) + \ldots + \rho_u det(C_{i,j+1,\ldots,j+u-1})] \\ &\times[\rho_1 det(C_{i+1,j+2,\ldots,j+u}) + \rho_2 det(C_{i+1,j+1,\ldots,j+u}) + \ldots + \rho_u det(C_{i+1,j+1,\ldots,j+u-1})] > 0. \end{aligned}$$

Therefore, (3) holds.

Sufficiency: From (3), we have

$$[\rho_1 det(C_{i,j+2,\ldots,j+u}) + \rho_2 det(C_{i,j+1,\ldots,j+u}) + \ldots + \rho_u det(C_{i,j+1,\ldots,j+u-1})] \neq 0.$$

Let

$$y_i = \mid \rho_1 det(C_{i,j+2,\ldots,j+u}) + \rho_2 det(C_{i,j+1,\ldots,j+u}) + \ldots + \rho_u det(C_{i,j+1,\ldots,j+u-1}) \mid \qquad \forall i \in H_m. \tag{5}$$

Because all entries of C are integers, all the determinants $det(C_{j_1,j_2,\ldots,j_u})$ are integers for all $1 \leq j_1, j_2, \ldots, j_u \leq m$. Evidently, each y_i is an integer and $y_i > 0$ for $\forall i \in H_m$. We will show that y_i is a solution of (4).

By (2) and (4), they satisfy

$$\sum_{i=1}^{m} c_{ij}\{(-1)^{i-1}[\rho_1 det(C_{i,j+2,j+3,\ldots,j+u}) + \rho_2 det(C_{i,j+1,j+3,\ldots,j+u}) + \ldots \\ \ldots + \rho_u det(C_{i,j+1,j+2,\ldots,j+u-1})]\} = 0 \tag{6}$$

or equivalently

$$c_{1j}\{\rho_1 det(C_{1,j+2,\ldots,j+u}) + \rho_2 det(C_{1,j+1,\ldots,j+u}) + \ldots + \rho_u det(C_{1,j+1,\ldots,j+u-1})\} \\ -c_{2j}\{\rho_1 det(C_{2,j+2,\ldots,j+u}) + \rho_2 det(C_{2,j+1,\ldots,j+u}) + \ldots + \rho_u det(C_{2,j+1,\ldots,j+u-1})\} + \ldots \\ \pm c_{mj}\{\rho_1 det(C_{m,j+2,\ldots,j+u}) + \rho_2 det(C_{m,j+1,\ldots,j+u}) + \ldots + \rho_u det(C_{m,j+1,\ldots,j+u-1})\} = 0. \tag{7}$$

From (3), there are the following cases:

Case 1): If

$$[\rho_1 det(C_{i,j+2,\ldots,j+u}) + \rho_2 det(C_{i,j+1,\ldots,j+u}) + \ldots + \rho_u det(C_{i,j+1,\ldots,j+u-1})] > 0$$

and $[\rho_1 det(C_{i+1,j+2,\ldots,j+u}) + \rho_2 det(C_{i+1,j+1,\ldots,j+u}) + \ldots + \rho_u det(C_{i+1,j+1,\ldots,j+u-1})] < 0$, $\forall i \in H_{m-1}$, in this case, it is clear that

$$[\rho_1 det(C_{1,j+2,\ldots,j+u}) + \rho_2 det(C_{1,j+1,\ldots,j+u}) + \ldots + \rho_u det(C_{1,j+1,\ldots,j+u-1})] > 0,$$

therefore

$$[\rho_1 det(C_{i,j+2,\ldots,j+u}) + \rho_2 det(C_{i,j+1,\ldots,j+u}) + \ldots + \rho_u det(C_{i,j+1,\ldots,j+u-1})] > 0,$$

if i is odd, and

$$[\rho_1 det(C_{i,j+2,\ldots,j+u}) + \rho_2 det(C_{i,j+1,\ldots,j+u}) + \ldots + \rho_u det(C_{i,j+1,\ldots,j+u-1})] < 0,$$

if i is even.

So equations (6) or (7) imply

$$\sum_{i=1}^{m} c_{ij}\{(-1)^{i-1}[\rho_1 det(C_{i,j+2,j+3,\dots,j+u}) + \rho_2 det(C_{i,j+1,j+3,\dots,j+u}) + \dots$$

$$\dots + \rho_u det(C_{i,j+1,j+2,\dots,j+u-1})]\} = 0.$$

Hence it is proved that (5) is a solution of (4). In other words there exists a positive integer vector $y = (y_1, y_2, \dots, y_m)^T$, such that $C^T y = 0$. Hence, the PN is conservative.

Case 2): If

$$[\rho_1 det(C_{i,j+2,\dots,j+u}) + \rho_2 det(C_{i,j+1,\dots,j+u}) + \dots + \rho_u det(C_{i,j+1,\dots,j+u-1})] < 0$$

and

$$[\rho_1 det(C_{i,j+2,\dots,j+u}) + \rho_2 det(C_{i,j+1,\dots,j+u}) + \dots + \rho_u det(C_{i,j+1,\dots,j+u-1})] > 0, \forall i \in H_{m-1},$$

in the same way, it is clear that

$$[\rho_1 det(C_{1,j+2,\dots,j+u}) + \rho_2 det(C_{1,j+1,\dots,j+u}) + \dots + \rho_u det(C_{1,j+1,\dots,j+u-1})] < 0.$$

Therefore

$$[\rho_1 det(C_{i,j+2,\dots,j+u}) + \rho_2 det(C_{i,j+1,\dots,j+u}) + \dots + \rho_u det(C_{i,j+1,\dots,j+u-1})] < 0,$$

if i is odd, and

$$[\rho_1 det(C_{i,j+2,\dots,j+u}) + \rho_2 det(C_{i,j+1,\dots,j+u}) + \dots + \rho_u det(C_{i,j+1,\dots,j+u-1})] > 0,$$

if i is even. The rest of the proof is similar to the above, except that we must multiply (6) or (7) by -1 in order to obtain (5). The proof is completed. □

Since $rank(C) = rank(C^T)$, the same arguments as in Theorem 4.4 can be used to prove the following corollaries on consistency. We consider the case: the rank of the incidence matrix C of the PN be $r = n - u$.

Corollary 4.5. Let r be the rank of the incidence matrix C of a PN, and let $\hat{C}^T$ and $C^T_{j+1,j+2,\dots,j+u}$, $j \in \{0, 1, \dots, r\}$ be as defined above. If $r = n - u$, $1 \le u < n$ then the PN is consistent if and only if there exist non-zero constants: $\rho_1, \rho_2, \dots, \rho_u$, such that:

$$[\rho_1 det(C^T_{i,j+2,\dots,j+u}) + \rho_2 det(C^T_{i,j+1,\dots,j+u}) + \dots + \rho_u det(C^T_{i,j+1,\dots,j+u-1})]$$

$$\times[\rho_1 det(C^T_{i+1,j+2,\dots,j+u}) + \rho_2 det(C^T_{i+1,j+1,\dots,j+u}) + \dots + \rho_u det(C^T_{i+1,j+1,\dots,j+u-1})] < 0$$

$$\forall i \in H_{n-1}. \tag{8}$$

P r o o f. The proof is similar to Theorem 4.1. □

Corollary 4.6. If it satisfies (3), then the PN is structurally bounded.

P r o o f . The proof is similar to Theorem 1 of [14]. □

Corollary 4.7. If it satisfied (8), then the PN is repetitive.

P r o o f . The proof is similar to Theorem 2 of [14]. □

In order to explain the significance and validity of our results, we give the two important remarks given as follows.

Remark 4.8. If $u = 1$, Theorem 4.4 and Corollary 4.5 are Proposition 1 and Corollary 1 of [14], respectively. For the integrality of this paper, we will list them.

Proposition 4.9. (Proposition 1 of [14]) Let $r = m - 1$ be the rank of the incidence matrix C of a PN, and let $\hat{C}$ and C_i be defined as above when $u = 1$. Then the necessary and sufficient condition for the PN to be conservative is that the following condition is fulfilled:

$$det(C_i)det(C_{i+1}) < 0, \forall i \in H_{m-1}. \tag{9}$$

Corollary 4.10. (Corollary 1 of [14]) Let $r = n-1$ be the rank of the incidence matrix C of a PN, and let $\hat{C}^T$ and C_i^T be defined as above when $u = 1$. Then the necessary and sufficient condition for the PN to be conservative is that the following condition is fulfilled:

$$det(C_i^T)det(C_{i+1}^T) < 0, \forall i \in H_{n-1}. \tag{10}$$

Note that Proposition 4.9 and Corollary 4.10 cover an important and a large class of PNs, for which the rank assumption holds such as the connected marked graph. Indeed, it is widely known that, for a connected marked graph with m nodes, the rank of the incidence matrix C is $m - 1$ (see [10]).

Remark 4.11. If $u = 2$, we obtain the two propositions, as follows:

Proposition 4.12. Let $r = m - 2$ be the rank of the incidence matrix C of a PN, and let $\hat{C}$ and C_{ij} be as defined above. Then the PN is conservative if and only if there exists two non-zero constants ρ_1 and ρ_2 such that:

$$[\rho_1 det(C_{i,k}) + \rho_2 det(C_{i,l})][\rho_1 det(C_{i+1,k}) + \rho_2 det(C_{i+1,l})] < 0, \forall i \in H_{m-1}, k, l \in H_m. \tag{11}$$

Corollary 4.13. Let $r = n - 2$ be the rank of the incidence matrix C of a PN, and let $\hat{C}^T$ and C_{ij}^T be as defined above. Then the PN is consistent if and only if there exist two nonzero constants ρ_1 and ρ_2 such that:

$$[\rho_1 det(C_{i,k}^T)+\rho_2 det(C_{i,l}^T)][\rho_1 det(C_{i+1,k}^T)+\rho_2 det(C_{i+1,l}^T)] < 0, \forall i \in H_{n-1}, k, l \in H_m. \tag{12}$$

Note that the value of every determinant $det(C_{i,k})$ of sub-matrix $C_{i,k}$ (i. e. $j_1 = i$, $j_2 = k$) can be computed and obtained by Matlab tools. The corresponding algorithm can be found in the Appendix.

Remark 4.14. All the results still hold, if C is replaced by product incidence matrix CC^T or C^TC in this section. The methods and techniques of the proofs are the same, except for computational complexity.

5. ILLUSTRATIVE EXAMPLE

In this section, we give two examples to illustrate our results.

Example 5.1. A simple resource allocation system is shown in Figure 3. Its incidence matrix C is

$$C = \begin{pmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ -2 & -3 & 5 \end{pmatrix}.$$

Then the PN is conservative and $C^T x = 0$ holds for $x = (4, 3, 1, 1, 1)^T$, see Example 4.3.

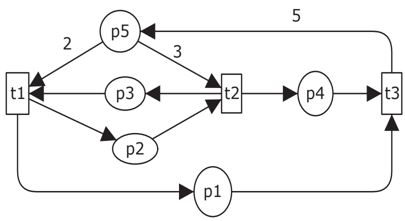

Fig. 3. A resource allocation system

Example 5.2. A system of simple sequential processes with multiple resources is shown in Figure 4. Its incidence matrix:

$$C^T = \begin{pmatrix} -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & -1 \end{pmatrix}.$$

Hence, $rank(C) = 6 = 8 - 2$. By the Appendix′s algorithm and $de(C_{45}) = 1 \neq 0$, we have $D_{8\times 2} =$

$$\begin{pmatrix} det(C_{15}) & det(C_{25}) & det(C_{35}) & det(C_{45}) & det(C_{55}) & det(C_{65}) & det(C_{75}) & det(C_{85}) \\ det(C_{14}) & det(C_{24}) & det(C_{34}) & det(C_{44}) & det(C_{54}) & det(C_{64}) & det(C_{74}) & det(C_{84}) \end{pmatrix}^T$$

that is,

$$D_{8\times 2} = \begin{pmatrix} -1 & 1 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & -1 & 1 \end{pmatrix}^T.$$

By Corollary 4.13, and taking $\rho_1 = \rho_2 = -1$, $k = 4, l = 5$, the PN is consistent and $Cy = 0$ holds for $y = (1, 1, 1, 1, 1, 1, 1, 1)^T$ given by Corollary 4.2.

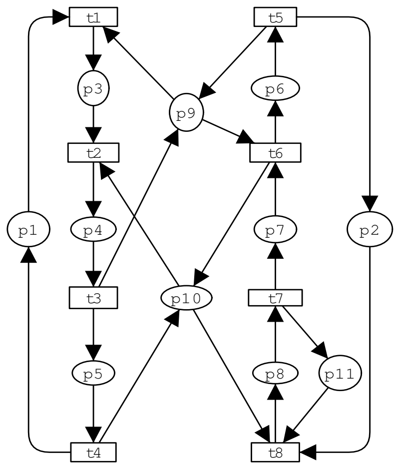

Fig. 4. A system of simple sequential processes with multiple resources.

6. CONCLUSION

A new technique is proposed in this paper to investigate the structural (partial) conservativeness and (partial) consistency of general Petri nets. First, by checking the reducibility and singularity of the product incidence matrix $A = CC^T$ and $\tilde{A} = C^TC$ and computing the adjugate matrix of the product incidence matrix, a sufficient condition is obtained to determine the well-structured of a Petri net. The matrix $C_{m\times n}$ denotes the incidence matrix of a Petri net, and whether it is a square matrix or not makes no difference to the results. In this way, the restrictions that the incidence matrix must be a square matrix in Ref. [14] can be avoided. The second step is to select some relevant rows of the incidence matrix which are linearly independent to build a matrix $\hat{C}$ by the rank of the incidence matrix, i. e. $rank(C) = r$. An algorithm is given to compute all proper minors of the matrix and a set of combinations of certain sub-determinants of the incidence matrix of the net can be obtained. By the signs of these combinations, we can easily determine the conservativeness and consistency of the net. These computations can be easily realized with MATLAB and LINDO software and the proposed technique can be well applied to general PNs.

APPENDIX

An algorithm is given to compute the determinants of the all sub-matrices $C_{j+1,j+2,\dots,j+u}$, $j \in \{0, 1, \dots, r\}$. The idea of the algorithm is simple. First, it searches $r = m - u$ rows of the incidence matrix C which are linearly independent by elementary transformation. Second, every $det(C_{j+1,j+2,\dots,j+u})$, $j \in \{0, 1, \dots, r\}$ are computed from big to small in the light of the lower index of $C_{j+1,j+2,\dots,j+u}$. If $det(C_{j+1,j+2,\dots,j+u}) \neq 0$, for some $j \in \{0, 1, \dots, r\}$, then all the values of the determinants $det(C_{j+1,j+2,\dots,j+u})$ are computed, for $\forall i \in H_m$, $j_1, j_2, \dots, j_u \in \{j + 1, j + 2, \dots, j + u\}$. Therefore, we obtain a matrix $D_{n\times u}$. Finally, we solve an ILPP, the values of the constants $\rho_1, \rho_2, \dots, \rho_u$ are obtained. Thus, we can confirm the conservativeness (or consistency) of the PN by Theorem 4.4 (or Corollary 4.5). The general idea can be summarized as follows:

Algorithm

Input: incidence matrix C.

Output: all the signs of β_i, $\forall i \in H_{m-1}$.

Step1: Confirming $u = m - r$ by computing the rank r of the incidence matrix C.

Step2: Choosing r rows of C, which are linearly independent, denoted as $\hat{C}$.

Step3: Computing $det(C_{i,j_1,j_2,\dots,j_{u-1}})$, for $\forall i \in H_{m-1}$, $j_k \in \{j+1, j+2, \dots, j+u\}$, $k = 1, \dots, u-1$, if $det(C_{j+1,j+2,\dots,j+u}) \neq 0$ for some $j \in \{0, 1, \dots, r\}$.

Step4: Obtaining a matrix $D_{n\times u}$, then we solve the ILPP:

Min $\{1_{1\times u} \cdot (\rho_1, \rho_2, \dots, \rho_u)^T\}$, s.t.

$$diag(1, -1, \dots, (-1)^{i-1}, 1, \dots, (-1)^{n-1}) \cdot D_{n\times u} \cdot (\rho_1, \rho_2, \dots, \rho_u)^T \geq 1_{n\times 1}$$

Step5: Checking if the values of

$$\begin{aligned} \beta_i = & [\rho_1 det(C_{i,j+2,j+3,\dots,j+u}) + \rho_2 det(C_{i,j+1,j+3,\dots,j+u}) + \dots \\ & \dots + \rho_u det(C_{i,j+1,j+2,\dots,j+u-1})] \\ & \times [\rho_1 det(C_{i+1,j+2,j+3,\dots,j+u}) + \rho_2 det(C_{i+1,j+1,j+3,\dots,j+u}) + \dots \\ & \dots + \rho_u det(C_{i+1,j+1,j+2,\dots,j+u-1})] \end{aligned}$$

are negative or not for $\forall i \in H_{m-1}$, $j \in \{0, 1, \dots, r\}$. If they are congruously negative, then the PN is conservative. Otherwise, the PN is not conservative. It is similar to the consistency of a PN.

To evaluate the computational complexity of the Step 4, we know the computational difficulty of an integer linear programming problems are determined by the number of integer variables and the structure of the problem. It is easy to infer that the unknowns are $\theta = u$ for the ILPP, which is a very simple optimization problem and is fast to solve using LINDO. Moreover, the computational complexity of the Step 3 is rested with the rank r of the incidence matrix of a PN and $u = n - r$ or $u = m - r$, i. e. the number of all determinants computed is ur.

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