Pablo Rocha; Marta Urciuolo

Errata of the paper “On the H^p-L^q boundedness of some fractional integral operators”

Czechoslovak Mathematical Journal, Vol. 64 (2014), No. 3, 867–868

Persistent URL: `http://dml.cz/dmlcz/144065`

Czechoslovak Mathematical Journal, 64 (139) (2014), 867–868

ERRATA OF THE PAPER "ON THE H^p-L^q BOUNDEDNESS OF SOME FRACTIONAL INTEGRAL OPERATORS"

Pablo Rocha, Marta Urciuolo, Córdoba

(Received February 18, 2014)

In the proof of Theorem 3.1 in [4], we state the following assertion:

If $0 \leqslant r < 1$, $0 < p \leqslant 1$, $1/q = 1/p - r$ *and* $f \in H^p(\mathbb{R}^n)$ *we write* $f = \sum_{j\in\mathbb{N}} \lambda_j a_j$, *where* a_j *is a* p*-atom and* $\sum_{j\in\mathbb{N}} |\lambda_j|^p \leqslant c\|f\|^p_{H^p}$. *So the theorem will be proved if we obtain that there exists* $c > 0$ *such that* $\|Ta\|_{L^q} \leqslant c$ *with* c *independent of the* p*-atom* a, *since this estimate and the inequality* $\Big(\sum_{j\in\mathbb{N}} |\lambda_j|^q\Big)^{1/q} \leqslant \Big(\sum_{j\in\mathbb{N}} |\lambda_j|^p\Big)^{1/p}$ *give* $\|Tf\|_q \leqslant c\|f\|_{H^p}$.

Although the final inequality holds, the assertion is not completely correct. Indeed, in [1], M. Bownik gives an example of a linear functional defined on a dense subspace of the Hardy space $H^1(\mathbb{R}^n)$ and he shows that although this functional is uniformly bounded on atoms, it does not extend to a bounded functional on the whole $H^1(\mathbb{R}^n)$. So in general it is not enough to verify that an operator or a functional is bounded on atoms to conclude that it extends boundedly to the whole space. See also [2].

By Proposition 2 in [4] we have that T is a well defined bounded operator from $L^s(\mathbb{R}^n)$ into $L^q(\mathbb{R}^n)$, $1/q = 1/s - r$, $1 < s < 1/r$. Also, from Remark 4.12 in [3], we obtain that the equality $f = \sum_{j\in\mathbb{N}} \lambda_j a_j$ holds in $L^s(\mathbb{R}^n)$ for $f \in H^p(\mathbb{R}^n) \cap L^s(\mathbb{R}^n)$. So, taking a subsequence if necessary, we get

$$|Tf(x)| \leqslant \sum_{j\in\mathbb{N}} |\lambda_j||T(a_j)(x)| \tag{1}$$

a.e. $x \in \mathbb{R}^n$.

So the correct assertion should be:

If $0 \leqslant r < 1$, $0 < p \leqslant 1$, $1/q = 1/p - r$, *taking* $1 < s < 1/r$ *and* $f \in H^p(\mathbb{R}^n) \cap L^s(\mathbb{R}^n)$ *we write* $f = \sum_{j\in\mathbb{N}} \lambda_j a_j$, *where* a_j *is a* p*-atom, the convergence* *is in* $H^p(\mathbb{R}^n)$ *and in* $L^s(\mathbb{R}^n)$, *with* $\sum_{j\in\mathbb{N}} |\lambda_j|^p \leqslant c\|f\|_{H^p}^p$. *So the theorem will be proved if we obtain that there exists* $c > 0$ *such that* $\|Ta\|_{L^q} \leqslant c$ *with* c *independent of the* p*-atom* a, *since this estimate,* (1) *and the inequality* $\Big(\sum_{j\in\mathbb{N}} |\lambda_j|^q\Big)^{1/q} \leqslant \Big(\sum_{j\in\mathbb{N}} |\lambda_j|^p\Big)^{1/p}$ *give* $\|Tf\|_q \leqslant c\|f\|_{H^p}$ *for* $f \in H^p(\mathbb{R}^n) \cap L^s(\mathbb{R}^n)$, *so the theorem follows from the density of* $H^p(\mathbb{R}^n) \cap L^s(\mathbb{R}^n)$ *in* $H^p(\mathbb{R}^n)$.

In [5], Theorem 1, we make a similar assertion. An analogous argument works.

References

[1] *M. Bownik*: Boundedness of operators on Hardy spaces via atomic decompositions. Proc. Am. Math. Soc. (electronic) *133* (2005), 3535–3542.

[2] *S. Meda, P. Sjögren, M. Vallarino*: On the H^1-L^1 boundedness of operators. Proc. Am. Math. Soc. *136* (2008), 2921–2931.

[3] *E. Nakai, Y. Sawano*: Hardy spaces with variable exponents and generalized Campanato spaces. J. Funct. Anal. *262* (2012), 3665–3748.

[4] *P. Rocha, M. Urciuolo*: On the H^p-L^q boundedness of some fractional integral operators. Czech. Math. J. *62* (2012), 625–635.

[5] *P. Rocha, M. Urciuolo*: On the H^p-L^p-boundedness of some integral operators. Georgian Math. J. *18* (2011), 801–808.

Authors' address: Pablo Rocha, Marta Urciuolo, Famaf-UNC, Ciem-Conicet, Medina Allende s/n, Ciudad Universitaria, 5000 Córdoba, Argentina, e-mail: `rp@famaf.unc.edu.ar`, `urciuolo@famaf.unc.edu.ar`.