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THE EFFECT OF A MAGNETIC FIELD ON THE ONSET OF BÉNARD CONVECTION IN VARIABLE VISCOSITY COUPLE-STRESS FLUIDS USING CLASSICAL LORENZ MODEL

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Abstract. The Rayleigh-Bénard convection for a couple-stress fluid with a thermorheological effect in the presence of an applied magnetic field is studied using both linear and non-linear stability analysis. This problem discusses the three important mechanisms that control the onset of convection; namely, suspended particles, an applied magnetic field, and variable viscosity. It is found that the thermorheological parameter, the couple-stress parameter, and the Chandrasekhar number influence the onset of convection. The effect of an increase in the thermorheological parameter leads to destabilization in the system, while the Chandrasekhar number and the couple-stress parameter have the opposite effect. The generalized Lorenz's model of the problem is essentially the classical Lorenz model but with coefficients involving the impact of three mechanisms as discussed earlier. The classical Lorenz model is a fifth-order autonomous system and found to be analytically intractable. Therefore, the Lorenz system is solved numerically using the Runge-Kutta method in order to quantify heat transfer. An effect of increasing the thermorheological parameter is found to enhance heat transfer, while the couple-stress parameter and the Chandrasekhar number diminishes the same.

Keywords: Rayleigh-Bénard convection; Boussinesq-Stokes suspension; variable viscosity; magnetoconvection; Lorenz model

MSC 2020: 76E30, 76W05

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Nomenclature

Latin symbols:

g	Acceleration due to gravity
d	Depth of the fluid layer
H_0	Reference magnetic field
p	Pressure
$\vec{q}$	Velocity vector (u, v)
Pr	Prandtl number
Pm	Magnetic Prandtl number
C	Couple stress parameter
Q	Chandrasekhar number
R_E	External thermal Rayleigh number
Nu	Nusselt number
T	Temperature
t	Time
ΔT	Temperature difference between lower and upper planes

Greek symbols:

$\pi\alpha$	Wave number
μ_f	Dynamic viscosity
μ'	Couple-stress viscosity
μ_m	Magnetic permeability
ν_m	Magnetic viscosity
β	Thermal expansion coefficient
χ	Constant thermal diffusivity
ϱ	Temperature-dependent density
ψ	Stream function
Ψ	Perturbed stream function
φ	Magnetic potential
Φ	Perturbed magnetic potential
Θ	Perturbed temperature
τ	Scaled time

Other symbols:

x, y, z	Cartesian coordinates
$\hat{i}$	Unit vector in x-direction
$\hat{k}$	Unit vector in z-direction
∇^2	Laplacian operator ($\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}$)

1. Introduction

In many practical situations, most fluids are not pure but may contain suspended particles like polymeric suspensions, liquid crystals, etc. Suspended particles play a prominent role in the analysis of a fluid. Due to the presence of suspended particles, there is a large stabilizing/destabilizing effect on the thermal convection. With the developing significance of couple-stress liquids in present-day science and innovation, investigations of such liquids are desirable. The hypothesis of couple-stress fluid got a more prominent consideration during recent years, as the conventional Newtonian liquids fail to depict all the attributes of the liquid flow with suspended particles. The study of such fluids has been a field of intensive research for the last few years, especially in many industrially important fluids like polymeric suspensions, chemicals, paints, solidifications of liquid crystals, and beverages, etc. The applications of challenging problems due to the presence of micro-sized suspended particles attract many researchers. Based on the size and concentration of suspended particles, the following mathematical models are available: Saffman's dusty gas model [15], the non-Newtonian fluid model [10], [4], [25], and references therein. The constitutive conditions for couple-stress liquids proposed by [25] demonstrate the least difficult hypothesis for micro-polar liquids that would allow for polar impacts, such as the presence of non-symmetric tensors, body couple, and couple stress. As indicated by the hypothesis proposed by the above study, couple-stresses are found to show up in recognizable sizes in liquids with huge particles, because the long chain of hyaluronic acidic atoms is present in synovial liquid as an added substance. The work [27] demonstrated synovial liquid in human joints to be a couple-stress liquid.

Rayleigh-Bénard convection problems with variable viscosity, heat source, and magnetic field have also drawn considerable attention in recent decades due to their applications in terrestrial planet convection problems and in many astrophysical and geophysical phenomena (see [9], [5], [19], and [18]). In many engineering and geophysical problems, viscosity strongly depends on temperature. The effect of strong variation of viscosity with temperature on the onset of natural convection by making use of Taylor series expansion has been investigated by [27], [2], [7], [17], [14], [24], and [26]. The experimental work on convection in fluids with thermorheological effect has been studied by [23]. The Rayleigh-Bénard convection in a square enclosure filled with viscoelastic fluid has been numerically investigated by [6]. Investigation [16] made a linear stability analysis of thermal convection in variable viscosity for a Newtonian ferromagnetic liquid by considering all possible boundary combinations in the presence of a heat source using Galerkin and shooting techniques. It is found that the effect of an increase in the variable viscosity parameter is to destabilize the system. The studies made by [8] and [21] demonstrates the effect of local thermal non-equilibrium (LTNE) on the onset of Brinkman-Bénard convection.

Magnetoconvection arises due to the interaction of an electrically conducting fluid with an external magnetic field leading to the Lorenz force. The investigation [11] is the non-linear stability analysis of Rayleigh Bénard magneto-convection under rotational speed modulation using the Ginzburg Landau model. They discussed the impact of various parameters in detail. The works carried out in [1], [2], [12], and [20] are based on temperature-dependent viscosity for a weak electrically conducting fluid under $1g$ and μg situations in the presence of the magnetic field. They made a detailed analysis of the impact of the thermorheological parameter and the Chandrasekhar number on the onset of stability for different boundary conditions.

Recently, the minimal representation of Fourier series in finite-amplitude analysis was mentioned in the study of chaotic thermal convection ([1], [17], and [22]). They made a detailed non-linear study of the Rayleigh-Bénard convection problem with some of the important mechanisms like a heat source, a magnetic field and the Coriolis force. It is found that these parameters influence the heat transfer coefficient the Nusselt number. The truncated Fourier series expansion has been found useful by many researchers for the following reasons:

(1) To make explicit the instability due to convection of many non-isothermal situations of practical interest.
(2) To perform linear stability analysis by obtaining an analytical expression for the thermal Rayleigh number and to quantify heat transfer.

The present study deals with an effect of Boussinesq-Stokes suspension, an external magnetic field, and temperature-dependent viscosity on the onset of stability and heat transfer due to the Rayleigh-Bénard convection. Here we considered the truncated Fourier cosine series for the basic viscosity and temperature gradient, and this is advantageous for the analytical solution. The Galerkin weighted residual technique is applied to derive the analytical expression for the thermal Rayleigh number, which is the eigenvalue of the problem. A weak non-linear study is made to demonstrate the influence of various parameters on the stability of convection and heat transfer.

2. Mathematical formulation

The schematic of the flow configuration consists of an electrically conducting Boussinesq-Stokes liquid between two infinitely extended horizontal planes $z = 0$ and $z = d$. A traverse magnetic field H_0 is uniformly applied in the vertical z-direction. Here the boundaries considered are stress-free and isothermal. The upper and lower boundaries are maintained at different temperatures T_0 and $T_0 + \Delta T$, respectively ($\Delta T > 0$) (Figure 1). Subjected to Oberbeck-Boussinesq approximation, the equations which govern the physical model of the problem involving electrically conducting couple-stress liquid are given below:

$$\nabla \cdot \vec{q} = 0, \tag{2.1}$$

$$\nabla \cdot \vec{H} = 0, \tag{2.2}$$

$$\begin{aligned}\varrho_0\Big(\frac{\partial \vec{q}}{\partial t} + (\vec{q}\cdot\nabla)\vec{q}\Big) &= -\nabla p - \varrho(T) g\hat{k} + \nabla(\mu_f(T)(\nabla\vec{q} + \nabla\vec{q}^{\,T})) \\ &\quad + \mu_m \vec{H}\cdot(\nabla\vec{H}) - \mu'\nabla^4\vec{q},\end{aligned} \tag{2.3}$$

$$\frac{\partial T}{\partial t} + (\vec{q}\cdot\nabla)T = \chi\nabla^2 T, \tag{2.4}$$

$$\frac{\partial \vec{H}}{\partial t} + (\vec{q}\cdot\nabla)\vec{H} = (\vec{H}\cdot\nabla)\vec{q} + \nu_m\nabla^2\vec{H}, \tag{2.5}$$

$$\varrho(T) = \varrho_0[1 - \beta(T - T_0)], \tag{2.6}$$

$$\mu_f(T) = \mu_0 \mathrm{e}^{-\delta(T - T_0)}. \tag{2.7}$$

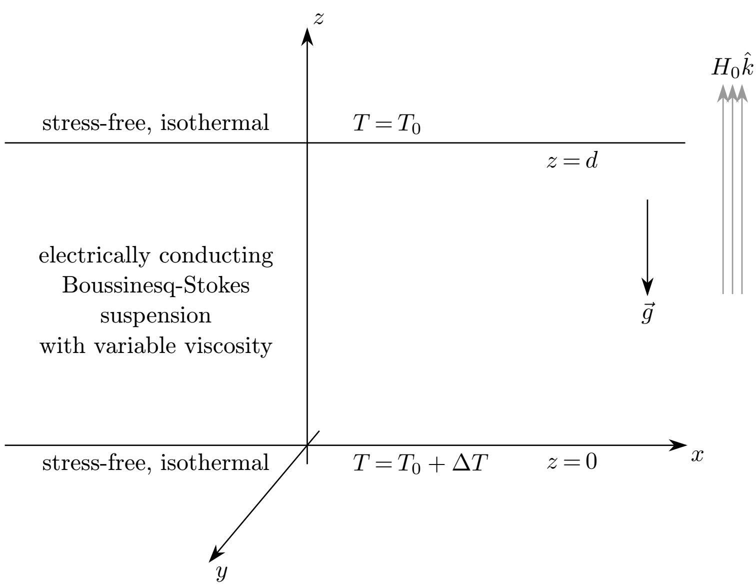

Figure 1. Physical model of the problem.

The viscosity-temperature model considered in (2.7) (see [10], [11], [13], [19]) represents the strong viscosity variation with temperature. Assuming the components of velocity $\vec{q}$, temperature T, density ϱ and dynamic viscosity μ_f in the basic state as $\vec{q}_b$, $T_b(z)$, $\varrho_b(z)$ and $\mu_{f_b}(z)$, the solutions of the governing equations in the basic quiescent state are of the form:

$$
(2.8)\qquad \begin{cases} \vec{q}_b = (0,0), \quad T_b = T_0 + \Delta T f\Big(\dfrac{z}{d}\Big), \quad \varrho_b\Big(\dfrac{z}{d}\Big) = \varrho_0\Big[1 - \beta\Delta T f\Big(\dfrac{z}{d}\Big)\Big], \\ \mu_{f_b}\Big(\dfrac{z}{d}\Big) = \mu_0 \mathrm{e}^{-Vf(z/d)}, \quad p_b\Big(\dfrac{z}{d}\Big) = -\displaystyle\int \varrho_b\Big(\dfrac{z}{d}\Big) g d\Big(\dfrac{z}{d}\Big) + k_1, \\ H_b\Big(\dfrac{z}{d}\Big) = k_2\Big(\dfrac{z}{d}\Big) + k_3, \end{cases}
$$

where $f(z/d) = 1 - z/d$, $V = \delta\Delta T$ is the thermorheological parameter, and k_1 is the constant of integration. We now superimpose the finite amplitude perturbations to the basic state in the form

$$
(2.9)\qquad \begin{cases} \vec{q} = \vec{q}_b(z) + \vec{q}\,'(x,z,t), \quad T = T_b(z) + T'(x,z,t), \quad \varrho = \varrho_b(z) + \varrho'(x,z,t), \\ p = p_b(z) + p'(x,z,t), \quad \vec{H} = \vec{H}_b(z) + \vec{H}'(x,z,t), \quad \mu = \mu_{f_b}(z) + \mu_f'(x,z,t), \end{cases}
$$

where the primes indicate its perturbed quantity. Substituting (2.9) into the governing equations, we get the following component equations:

$$
(2.10)\qquad \nabla \cdot \vec{q}\,' = 0,
$$

$$
(2.11)\qquad \nabla \cdot \vec{H}' = 0,
$$

$$
(2.12)\qquad \varrho_0\Big(\frac{\partial \vec{q}\,'}{\partial t} + (\vec{q}\,' \cdot \nabla)\vec{q}\,'\Big) = -\nabla p' - \varrho'(T) g\hat{k} + \nabla(\mu_f(T)(\nabla\vec{q}\,' + \nabla\vec{q}^{\,T})) - \mu_m^2(\vec{H}' \cdot \nabla)\vec{H}' + \mu_m H_b \frac{\partial \vec{H}'}{\partial z} - \mu'\nabla^4\vec{q}\,',
$$

$$
(2.13)\qquad \frac{\partial T'}{\partial t} + (\vec{q}\,' \cdot \nabla)T' + w'\frac{\partial T_b}{\partial z} = \chi\nabla^2 T',
$$

$$
(2.14)\qquad \frac{\partial \vec{H}'}{\partial t} + (\vec{q}\,' \cdot \nabla)\vec{H}' - (\vec{H}' \cdot \nabla)\vec{q}\,' - H_b\frac{\partial w'}{\partial z} = \nu_m\nabla^2\vec{H}',
$$

$$
(2.15)\qquad \varrho' = -\varrho_0\beta T'.
$$

Here, we restrict our study to only two dimensions, and therefore we introduce the magnetic potential φ' and the stream function ψ' as follows:

$$
(2.16)\qquad \begin{cases} u' = -\dfrac{\partial \psi'}{\partial z}, \quad w' = \dfrac{\partial \psi'}{\partial x}, \\ H_x' = -\dfrac{\partial \varphi'}{\partial z}, \quad H_z' = \dfrac{\partial \varphi'}{\partial x}. \end{cases}
$$

The classical procedure of operating curl for (2.12) helps in eliminating the pressure p, and transforming the above system of equations to dimensionless form using the scaling mentioned below:

$$(X,Z)=\Big(\frac{x}{d},\frac{z}{d}\Big),\quad \Psi=\frac{\psi'}{\chi},\quad \Theta=\frac{T'}{\Delta T},\quad \Phi=\frac{\varphi'}{dH_0}, \tag{2.17}$$

we obtain the non-dimensional governing equations as

$$\begin{aligned}\frac{1}{\mathrm{Pr}}\Big(\frac{\partial}{\partial t}(\nabla^2\Psi)+J(\Psi,\nabla^2\Psi)\Big)\\ &= R_E\frac{\partial\Theta}{\partial X}-C\nabla^6\Psi+Q\,\mathrm{Pm}\Big(\frac{\partial(\nabla^2\Phi)}{\partial Z}+J(\Phi,\nabla^2\Phi)\Big)\\ &\quad+\mu_{f_b}\nabla^4\Psi+\frac{\partial\mu_{f_b}}{\partial Z}\frac{\partial}{\partial Z}(\nabla^2\Psi),\end{aligned} \tag{2.18}$$

$$\Big(\frac{\partial}{\partial t}-\nabla^2\Big)\Theta=\frac{\partial\Psi}{\partial X}+J(\Psi,\Theta), \tag{2.19}$$

$$\Big(\frac{\partial}{\partial t}-\mathrm{Pm}\,\nabla^2\Big)\Phi=\frac{\partial\Psi}{\partial Z}+J(\Psi,\Phi), \tag{2.20}$$

where $\mathrm{Pr}=\mu/(\varrho_0\chi)$ is the Prandtl number, $\mathrm{Pm}=\nu_m/\chi$ is the magnetic Prandtl number, $R_E=\beta\varrho_0 gd^3\Delta T/(\mu_0\chi)$ is the thermal Rayleigh number, $Q=\mu_m^2\sigma H_0^2 d^2/\mu$ is the Chandrasekhar number, and $C=\mu'/(\mu d^2)$ is the couple stress parameter. Also, $J(F_1,H_1)=(\partial F_1/\partial X)(\partial H_1/\partial Z)-(\partial F_1/\partial Z)(\partial H_1/\partial X)$, $\nabla^2=\partial^2/\partial X^2+\partial^2/\partial Z^2$ is the Laplace operator. Also, the quantity $\mu_{f_b}(Z)=\mathrm{e}^{-V(1-Z)}$ is expressed as a half-range Fourier cosine series in the interval $[0,1]$, and this representation helps in obtaining the analytical expression for the thermal Rayleigh number. Moreover, the truncated Fourier cosine series is good enough to represent the basic viscosity. It is even clear that the basic states of the temperature-gradient and viscosity are linear.

The boundary conditions for the present problem on velocity, temperature, and magnetic potential are

$$\begin{cases}\dfrac{\partial\Psi}{\partial X}=\dfrac{\partial^2\Psi}{\partial Z^2}=\Theta=\dfrac{\partial\Phi}{\partial Z}=0 \quad \text{at } Z=0,\\[2ex] \dfrac{\partial\Psi}{\partial X}=\dfrac{\partial^2\Psi}{\partial Z^2}=\Theta=\dfrac{\partial\Phi}{\partial Z}=0 \quad \text{at } Z=1.\end{cases} \tag{2.21}$$

In the case of a free-free surface, the boundary conditions on the velocity of the fluid depend on whether surface tension is present or not. If the free surface does not deform in the direction normal to itself, we must require that $w=0$ at the boundaries. In the case where the surface tension is absent, the condition on velocity at the free surface is $w=\mathrm{d}^2w/\mathrm{d}z^2=0$. This condition is called the stress free condition. If the bounding wall of the fluid layer has high heat conductivity and heat capacity, in this case the temperature, would be uniform and unchanging in time, i.e., the boundary temperature would be unperturbed by any flow or temperature perturbations in the fluid. Thus $T = 0$ at the boundaries. This boundary condition is known as isothermal or a boundary condition of the first kind. This condition is also known to be the Dirichlet condition.

2.1. Linear theory. In ordered to perform linear analysis, we consider a linearized version of (2.18), (2.19), and (2.20) along with the stress free and isothermal boundary conditions (2.21). For the linear theory, it is essential to neglect the Jacobians, i.e., $J(\Psi, \nabla^2\Psi)$, $J(\Phi, \nabla^2\Phi)$, $J(\Psi, \Theta)$ and $J(\Psi, \Phi)$ in (2.18), (2.19) and (2.20) as it removes the products of amplitudes, which are quite small. Moreover, the products of these amplitudes have negligible impact on linear stability analysis. The solution of these equations are assumed to be periodic waves (see [3]) in the following form:

$$\begin{cases} \Psi(X,Z) = \dfrac{\sqrt{2}\eta_1^2}{\pi^2\alpha}\Psi_0 \sin(\pi\alpha X)\sin(\pi Z), \\ \Theta(X,Z) = \dfrac{\sqrt{2}\eta_1^2 c_1}{\pi^2\alpha c_2}\Theta_0 \cos(\pi\alpha X)\sin(\pi Z), \\ \Phi(X,Z) = \dfrac{\eta_1^2 c_1}{\sqrt{2}\alpha c_2}\Phi_0 \sin(\pi\alpha X)\cos(\pi Z), \end{cases} \tag{2.22}$$

where

$$\eta_1^2 = \pi^2(1+\alpha^2), \quad c_1 = \mathrm{Pr}\Big(\frac{\eta_1^2 - 2\pi^2}{2\eta_1^2}a_2 - \frac{a_0}{2}\Big) - C\eta_1^2 \quad \text{and} \quad c_2 = \frac{\mathrm{Pr}\,\pi\alpha R_E}{\eta_1^4}.$$

The analytical expression for the thermal Rayleigh number using the Galerkin technique is obtained as

$$R_E = \frac{\eta_1^2(Q\pi^2 + C\eta_1^6 - \frac{1}{2}\eta_1^4((1-2\pi^2)a_2 - a_0))}{\pi^2\alpha^2}, \tag{2.23}$$

where $a_0 = 2\int_0^1 \mu_{f_b}\,\mathrm{d}Z$ and $a_2 = 2\int_0^1 \mu_{f_b}\cos(2\pi Z)\,\mathrm{d}Z$ are the Fourier cosine coefficients and $\pi\alpha$ is the horizontal wave number. The quantities Ψ_0, Θ_0, and Φ_0 are, respectively, the amplitudes of the stream function, temperature, and magnetic potential. In order to derive the above expression for R_E, we substitute (2.22) into the linearized version of the dimensionless equations (2.18)–(2.20), and then applying the standard Galerkin procedure (i.e. integrating the equations with respect to X and Z between $[0, 2/\alpha]$ and $[0, 1]$, respectively), we obtain the set of homogeneous linear system of equations in Ψ_0, Θ_0, and Φ_0. In the event of obtaining a non-trivial solution of the linear system, the expression for R_E is derived.

Linear theory explains only the conduction to the convective state of the system, but it fails in analyzing the heat transfer. We now embark upon the weak non-linear theory which demonstrates quantification of the heat transfer.

2.2. Non-linear stability analysis using minimal Fourier modes. The minimal representation of double Fourier series for the stream function Ψ, temperature Θ and the magnetic potential Φ is given by

$$\left\{\begin{aligned} \Psi(X,Z,\tau) &= \frac{\sqrt{2}\eta_1^2}{\pi^2\alpha}A(\tau)\sin(\pi\alpha X)\sin(\pi Z),\\ \Theta(X,Z,\tau) &= \frac{\sqrt{2}\eta_1^2 c_1}{\pi^2\alpha c_2}B(\tau)\cos(\pi\alpha X)\sin(\pi Z) - \frac{\eta_1^2 c_1}{\pi^2\alpha c_2}D(\tau)\sin(2\pi Z),\\ \Phi(X,Z,\tau) &= \frac{\eta_1^2 c_1}{\sqrt{2}\alpha c_2}E(\tau)\sin(\pi\alpha X)\cos(\pi Z) - \frac{\eta_1^2 c_1}{2\alpha c_2}F(\tau)\sin(2\pi\alpha X).\end{aligned}\right. \tag{2.24}$$

In the above expressions, the time-dependent amplitudes $A(\tau)$, $B(\tau)$, $D(\tau)$, $E(\tau)$, and $F(\tau)$ are to be determined from the dynamics of the system. Substitute these into (2.18)–(2.20), and then performing standard orthogonalization procedure yields the following classical Lorenz model of fifth-order:

$$\frac{\mathrm{d}A}{\mathrm{d}\tau} = \frac{\pi^2\alpha}{\sqrt{2}\eta_1^2}\Big(c_1(B-A) - \frac{\pi^2 c_1 c_3}{2c_2}E - \frac{\pi^2\eta_1^2 {c_1}^2 c_4}{4\alpha {c_2}^2}EF\Big), \tag{2.25}$$

$$\frac{\mathrm{d}B}{\mathrm{d}\tau} = r_f A - B - AD, \tag{2.26}$$

$$\frac{\mathrm{d}D}{\mathrm{d}\tau} = AB - bD - AE, \tag{2.27}$$

$$\frac{\mathrm{d}E}{\mathrm{d}\tau} = AF - 2AD - (1+2b)E, \tag{2.28}$$

$$\frac{\mathrm{d}F}{\mathrm{d}\tau} = 2AE - 4bF, \tag{2.29}$$

where

$$c_3 = \frac{\mathrm{Pm}\,Q\pi}{\eta_1^2}, \quad c_4 = \mathrm{Pm}\,Q\Big(\frac{4\pi^4\alpha^3 - 2\pi^2\alpha\eta_1^2}{\eta_1^4}\Big), \quad r_f = \frac{c_2\,\pi\alpha}{c_1\eta_1^2}$$

and

$$b = \frac{\mathrm{Pm}\,\pi^2\alpha^2}{\eta_1^2}.$$

The analytical solution for the generalized Lorenz model derived is not possible due to the presence of general time dependent variable. Hence, these equations are solved using a numerical procedure.

3. Heat transfer using the classical Lorenz model

As convection set in with a sufficient temperature gradient across the fluid layer, it can be identified by its effect on the heat transport. The horizontally averaged Nusselt number Nu at the lower boundary for the stationary mode of magnetoconvection is given by

$$\mathrm{Nu}(\tau) = \frac{\left[\frac{1}{2}\alpha_c\left(\int_{X=0}^{2/\alpha}(1-Z+\Theta)_Z \,\mathrm{d}X\right)\right]_{Z=0}}{\left[\frac{1}{2}\alpha_c\left(\int_{X=0}^{2/\alpha}(1-Z)_Z \,\mathrm{d}X\right)\right]_{Z=0}}. \tag{3.1}$$

Substitute (2.24) into (3.1) and integrate, the expression $\mathrm{Nu}(\tau)$ is obtained as:

$$\mathrm{Nu}(\tau) = 1 + \frac{2c_1{\eta_1}^2}{c_2\pi\alpha}D(\tau). \tag{3.2}$$

The second term on the right-hand side of the above equation signifies convective heat transfer, while the first term signifies the conduction state.

4. Results and discussion

This study demonstrates the Rayleigh-Bénard convection problem in an electrically conducting fluid layer under the influence of suspended particles, the temperature-dependent viscosity, and the applied vertical magnetic field. These parameters have appeared in the form of the couple-stress parameter C, the thermorheological parameter V, and the Chandrasekhar number Q respectively. The effects of electrical conductivity and the magnetic field come through the parameters Pm and Q. The effect of all these parameters on the onset of stability and convective heat transfer is discussed in detail.

4.1. Linear theory. A few significant highlights of linear stability analysis are:

(1) Obtaining the half range Fourier cosine series for the basic-viscosity and the temperature gradient in the interval $[0, 1]$.
(2) Finding an analytical expression for the eigenvalue R_E in terms of Q, C, and V using Galerkin technique.
(3) Plotting neutral stability curves (Rayleigh-wave number graphs) to understand the impact of these parameters on stability.

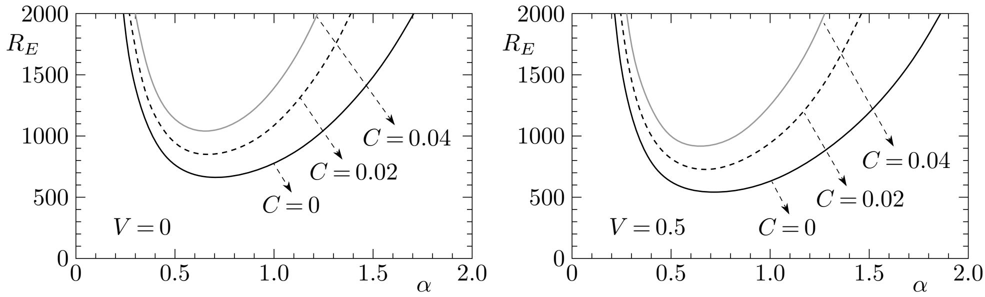

Figure 2. Plot of R_E vs. α for $Q = 0$.

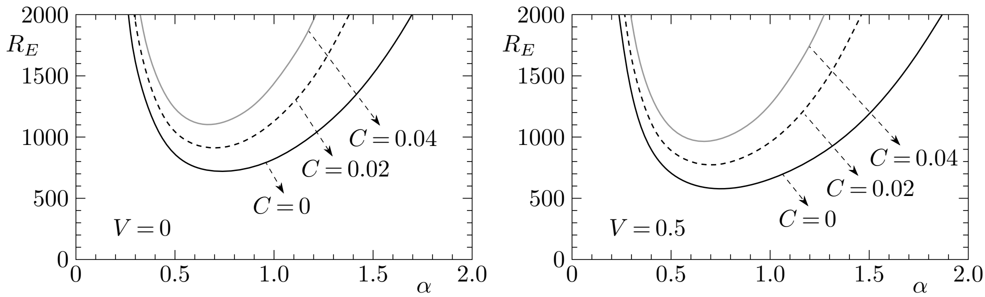

Figure 3. Plot of R_E vs. α for $Q = 2$.

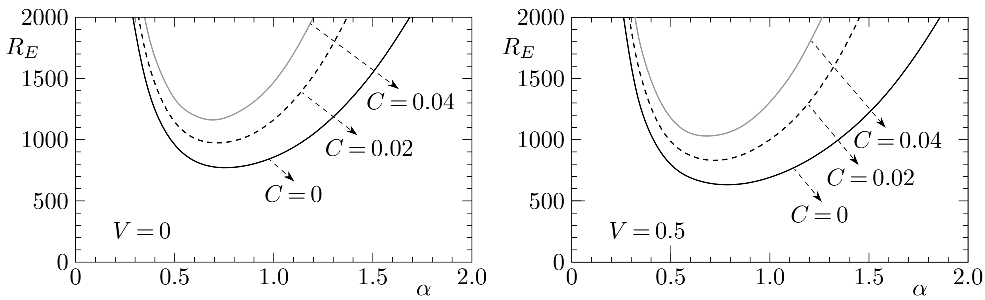

Figure 4. Plot of R_E vs. α for $Q = 4$.

The Figures 2–4 represent the variation of the thermal Rayleigh number R_E versus the wavenumber α under the influence of Q, V and C. It can be observed that both R_{E_c} and α_c increases with the increasing Q, which shows the stabilizing effect in the presence of the magnetic field. It is also apparent from these plots that as C increases, the R_{E_c} and α_c increases. Therefore, the system is more stable in the presence of suspended particles than the magnetic field. Also an effect of increasing the thermorheological parameter V is to decrease R_{E_c} and α_c, hence destabilizing the system.

4.2. Non-linear theory. A few significant highlights of non-linear stability analysis are:

(1) A finite amplitude perturbation procedure is employed through double Fourier series representation for the magnetic potential Φ, the stream function Ψ and the temperature field Θ.
(2) The non-linear system of equations is obtained in the form of a generalized Lorenz model and hence transformed to the classical model.
(3) Quantification of heat transport is made through the Nusselt number for the stationary mode of magnetoconvection in the Boussinesq-Stokes suspension fluid.
(4) Nusselt-time plots are made in order to study the individual effects of the parameters involved.

Before we begin the discussion of the model, we note here that the Lorenz model of the problem is numerically solved by employing a Runge-Kutta method with the adaptive step size. In order to carry out the numerical solution of the coupled system (2.25)–(2.29), we consider the initial conditions $A(0) = B(0) = D(0) = E(0) = F(0) = 5$. However, for any other initial conditions, the dynamics of the Lorenz system will be the same. Due to the suspended particles being present in the carrier liquid, the Prandtl number of the couple-stress liquid is more than that of the Newtonian carrier liquid. Hence, we have chosen $\mathrm{Pr} = 10$ and this is maintained throughout the discussion. Figures 5–7 depict the variation of the Nusselt number Nu with respect to the time τ under the influence of Q, C, and V. The individual effects of the magnetic field, suspended particles and temperature dependent viscosity can be observed from each of the Nusselt number plots. Figure 5 highlights the effect of variable viscosity in the absence of a magnetic field Q. The Nusselt number Nu increases with increasing values of V, indicating the convective contribution to heat transport.

A magnetic field is one of the important external mechanisms that can be effectively used to control the onset of convection. The effect of the magnetic field by means of Chandrasekhar number Q on a heat transfer coefficient is observed in Figures 5 and 6. From 5 (a) and 6 (a), we notice the diminishing effect of heat transport due to the enhancement of the magnetic field. Hence, the increase of Q corresponds to a decrease in Nusselt number Nu. From all these plots, it is also clear that Nu decreases with increasing values of Q and C. The presence of suspended particles a larger temperature gradient is required to make the system unstable which resulting in a stabilizing effect of C.

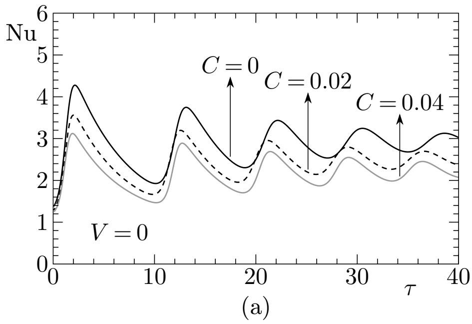

Figure 5. Nusselt number Nu variation plot with τ for $Q = 0$, $\Pr = 10$.

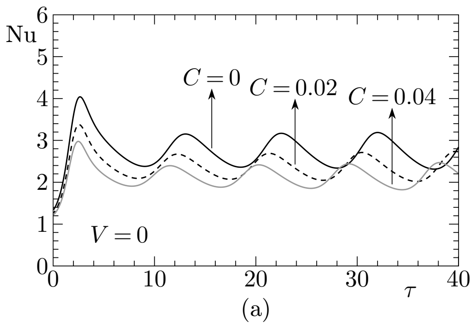

Figure 6. Nusselt number Nu variation plot with τ for $Q = 2$, $\Pr = 10$.

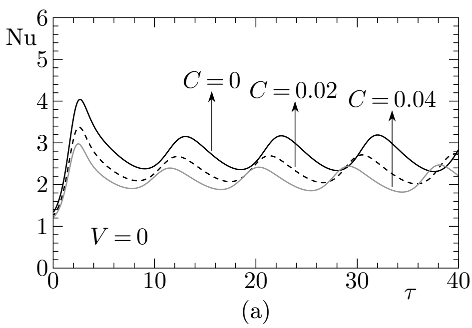

Figure 7. Nusselt number Nu variation plot with τ for $Q = 4$, $\Pr = 10$.

5. Conclusion

This paper presents a weakly non-linear study of Rayleigh-Bénard convection of Boussinesq-Stokes suspension fluid in the presence of an applied magnetic field and temperature-dependent viscosity. An analytical expression for the critical Rayleigh number R_E is derived as the function of C, V, and Q. The linear study concludes that the effect of the magnetic field and suspended particles is to stabilize the system whereas the effect of variable viscosity destabilizes the same. Non-linear stability analysis involves the derivation of the classical Lorenz model and its numerical solution to find out the amplitudes required to quantify heat transport. From $\mathrm{Nu}-\tau$ plots, we conclude that increasing the values of Q and C decreases the Nusselt number, which proves that a magnetic field and suspended particles reduce the heat transfer in the system. The effect of increasing the thermorheological parameter is to enhance the heat transfer. From all these plots, the following results are also apparent:

$$\mathrm{Nu}\,|_{Q=2} > \mathrm{Nu}\,|_{Q=4}, \quad \mathrm{Nu}\,|_{C=0.02} > \mathrm{Nu}\,|_{C=0.04}, \quad \mathrm{Nu}\,|_{V=0} < \mathrm{Nu}\,|_{V=0.5}.$$

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