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NEURAL NETWORK-BASED FAULT DIAGNOSIS AND FAULT-TOLERANT CONTROL FOR NONLINEAR SYSTEMS WITH OUTPUT MEASUREMENT NOISE

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In this article, the problems of fault diagnosis (FD) and fault-tolerant control (FTC) are investigated for a class of nonlinear systems with output measurement noise. Due to the influence of measurement noise in the output sensor, the output observation error cannot be accurately obtained, which causes obstacles to the accuracy of FD. To address this issue, an output filter and disturbance estimator are constructed to decrease the negative effects of measurement noise and observer gain disturbances, and a novel non-fragile neural observer is designed to estimate the unknown states. A new evaluation function is also introduced to detect faults. Then, a novel neural FTC controller is proposed in the presence of faults, to ensure that all the closed-loop system signals are semiglobally uniformly ultimately bounded (SGUUB). The effectiveness of the proposed methodology is verified via numerical simulation of a one-link robot system.

Keywords: fault diagnosis, fault-tolerant control, output measurement noise, non-fragile, output filter

Classification: 93C10, 94C12

1. INTRODUCTION

The fault-tolerant control (FTC) has received great attention in recent years because faults often occur in real engineering systems, which may lead to severe economic damages. FTC is a technology that improves the safety and reliability of a system, and has high significance for compensating for faults in real-time online. In general, FTC methods are usually divided into two categories, i. e., the passive FTC [4, 9, 20, 33] and the active FTC [5–7, 10, 11, 22, 28, 29, 31, 32]. The passive FTC is considered as a robustness control operation, but its adaptive ability to fault tolerance is limited. The active FTC is to readjust the controller's parameters after faults have occurred based on fault conditions. Neural networks (NNs) have been widely used to solve the active FTC problems for nonlinear systems with faults due to their approximate ability to unknown functions [17, 19, 21, 31, 32, 36, 37, 39]. In [31], neural networks were used to research the active FTC problems for a class of switching nonlinear systems. The article [29] studied the problem of adaptive active FTC for a class of nonlinear systems with an unidentified

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actuator fault. At present, the application of the active FTC in nonlinear systems has made great achievements. However, few results have been derived for nonlinear systems when the output sensor is affected by noise.

In practical engineering applications, systems and sensors are usually inevitably affected by disturbance and noise [11, 12, 15, 30, 35]. Noise may be caused by unforeseen operating conditions or component performance, such as nonlinear drifts and failures of electronic components. The performance of a nonlinear system may be negatively affected if the impact of random noise and disturbances is ignored. Moreover, the active FTC requires fault diagnosis (FD) to obtain fault information. Faults are usually diagnosed by the threshold of the output observation error. The authors in [31] constructed an evaluation function based on the output observation error. In [23], an active fault-tolerant control framework was specifically proposed for time-varying actuator faults to enhance the robustness of fault detection. The authors in [8] investigated the problem of FTC for multi-agent systems with sensor faults.

If the output sensor contains measurement noise, the output observation error will not be accurately captured. Then, the FD scheme may be ineffective. Therefore, for a system with output measurement noise, it is worthwhile to investigate the ability of an FD scheme to tolerate the noise. In addition, gains of observers may drift [3, 16, 18, 30] because of the unavoidable presence of truncation errors in numerical calculations or the aging of sensor equipment. In practice, besides the influence of output measurement noise, observer gain disturbances have also an impact on the accuracy of FD. Such disturbances can potentially lead to instability of the closed-loop system. Presently, majority of research on non-fragile observers design predominantly revolves around the utilization of linear matrix inequality (LMI) technology [13, 25–27, 34, 40]. While computer-based verification allows for convenience of LMI conditions, the design methods will become ineffective if the solvability conditions are not met. Especially, when there exit time-varying matrix inequalities, the complexity is further heightened. Thus, it is a formidable challenge to explore alternative approaches of non-fragile observers to effectively attenuate the impact of observer gain disturbances.

Based on the above analysis, this paper investigates a non-fragile FTC strategy for nonlinear systems with output measurement noise. The fault-tolerant scheme is activated by evaluating the FD condition to determine the occurrence of faults, and the non-fragile FTC is triggered when faults are detected. The main results of this paper can be summarized as follows: 1. By introducing an output filter, an extended system is derived and a non-fragile observer is constructed to overcome the adverse effects on FD accuracy caused by output measurement noise and observer gain perturbations. 2. Base on the output observation error threshold of the extended system, a novel FD function is proposed to enhance the reliability of diagnostic result. Then, the FTC scheme will be activated when necessary, and ensure that all signals of the closed-loop system are SGUUB.

The remainder of this paper is organized as follows. The description of the problem and some assumptions are presented in Section 2. In section 3, fault diagnosis and fault-tolerant control are addressed for a class of nonlinear systems with output measurement noise. In Section 4, the validity of the method is verified through a robotic arm simulation example. Section 5 gives the conclusions of the paper.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider a class of nonlinear system with fault as follows

$$\begin{cases} \dot{x}_1 = x_2 + f_1(\bar{x}_1) + d_{\sigma 1}(t), \\ \vdots \\ \dot{x}_n = u(t) + f_n(\bar{x}_n) + \beta(t - t_x)\,\eta(\bar{x}_n) + d_{\sigma n}(t), \\ y = x_1 + d_{\sigma 0}(t), \end{cases} \tag{1}$$

where, for $i = 1, \ldots, n$, $\bar{x}_i = (x_1, \ldots, x_i)^T \in \mathbb{R}^i$ are the system state vectors, $u(t)$ is the control input of the system, y denotes the system output, $d_{\sigma 0}(t)$ is a bounded unknown measurement noise in the sensor, $d_{\sigma i}(t)$ are bounded time-varying system disturbances, $f_i(\bar{x}_i)$ are unknown continuous functions which satisfy the following assumption.

Assumption 2.1. The nonlinear functions $f_i(\bar{x}_i), i = 1, 2, \ldots, n$, satisfy the following conditions

$$\left|f_i(\bar{x}_i) - f_i\left(\hat{\bar{x}}_i\right)\right| \leqslant q_i\left(|x_1(t) - \hat{x}_1(t)| + |x_2(t) - \hat{x}_2(t)| + \cdots + |x_i(t) - \hat{x}_i(t)|\right), \tag{2}$$

for any x_i, $\hat{\bar{x}}_i = (\hat{x}_1, \ldots, \hat{x}_i)^T \in \mathbb{R}$, where q_i are real numbers.

In this paper, we only consider the sudden fault. $\eta(\bar{x}_n)$ represents a fault function, t_x indicates an unknown time instant when a fault occurs, and $\beta(t - t_x)$ denotes the time profile of the fault in the following form

$$\beta(t - t_x) = \begin{cases} 0, \text{ if } t < t_x, \\ 1, \text{ if } t \geqslant t_x. \end{cases} \tag{3}$$

A non-zero FD delay between fault occurrence and diagnosis is unavoidable due to the existence of response time for FD, i. e. $t_d > t_x$, where t_d is the time when the FTC scheme is activated. In order to guarantee the control performance within a non-zero FD delay time $t \in [t_x, t_d]$, the fault function should satisfy the following assumption.

Assumption 2.2. (Zhao and Polycarpou [38]) For $t \in [t_x, t_d]$, the fault function $\eta(\bar{x}_n(t))$ satisfies $\|\eta(\bar{x}_n(t))\| \leqslant v$, where v is an unknown positive constant.

The following lemma can be found in [14], and is useful for our main results.

Lemma 2.3. (Koo and Choi [14]) Define two matrices $\tilde{A} \in \mathbb{R}^{(n+1)\times(n+1)}$ and $P_0 \in \mathbb{R}^{(n+1)\times(n+1)}$ as,

$$\tilde{A} = \begin{bmatrix} -\kappa_{n+1} & 1 & 0 & \cdots & 0 \\ -\kappa_n \eta_n(t) & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\kappa_2 \eta_2(t) & 0 & 0 & \cdots & 1 \\ -\kappa_1 \eta_1(t) & 0 & 0 & \cdots & 0 \end{bmatrix}, \ P_0 = \begin{pmatrix} a_{1\ell} & 0 & 0 & \cdots & 0 & 0 \\ -b_{2\ell} a_{1\ell} & a_{2\ell} & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & a_{n\ell} & 0 \\ 0 & 0 & 0 & \cdots & -b_{n+1,\,\ell} a_{n\ell} & a_{n+1,\,\ell} \end{pmatrix},$$

where $\eta_i(t)$, $i = 1, 2, \ldots, n$ are unknown continuous functions which satisfy the conditions $\eta_i^{\min} \leqslant \eta_i(t) \leqslant \eta_i^{\max}$, $a_{j\ell}$, $j = 1, 2, \ldots, n$ and $b_{j\ell}$, $j = 2, \ldots, n+1$ are positive constants, and κ_i, $i = 1, 2, \ldots, n+1$ are the observer gains.

The positive constants $a_{j\ell}$, $j = 1, \ldots, n+1$ can be obtained by

$$\begin{cases} a_{n+2,\,\ell} = 1, \\ a_{n+1,\,\ell} = c_1, \\ a_{n\ell} = \frac{2a_{n+1,\,\ell}}{b_{n+1,\,\ell}}\kappa_0, \\ a_{i-1,\,\ell} = \dfrac{2a_{i\ell}}{b_{i\ell}}\Bigg(\kappa_0 + \dfrac{ia_{i\ell}}{2a_{i+1,\,\ell}}b_{i+1,\,\ell} + \dfrac{ia_{i\ell}}{2a_{i+1,\,\ell}b_{i+1,\,\ell}} \\ \qquad + \dfrac{1}{2}\displaystyle\sum_{j=i}^{n}\Bigg(\left(\dfrac{b_{j+2,\,\ell}a_{j+1,\,\ell}}{a_{j+2,\,\ell}} + \dfrac{jb_{j+1,\,\ell}a_{j\ell}}{a_{j+1,\,\ell}}\right)\prod_{\kappa=i+1}^{j+1} b_{\kappa\ell}^2\Bigg)\Bigg), \; i = 2, \ldots, n, \end{cases}$$

where c_1 and κ_0 are two arbitrary positive constants, and $b_{n+2,\,\ell} = 0$.

The positive constants $b_{j\ell}$, $j = 2, \ldots, n+1$ satisfy the following conditions

$$\begin{cases} \left(n^2 \prod_{\kappa=2}^{j} b_{\kappa\ell} \max\left\{\left(\eta_{n-i+2}^{\max} - \eta_{n-i+3}^{\min}\right)^2, \left(\eta_{n-i+3}^{\max} - \eta_{n-i+2}^{\min}\right)^2\right\}\right) \\ (\alpha_1(\cdot) + \alpha_2(\cdot) + \alpha_3(\cdot)) < 1, \; j = 2, \ldots, n+1, \\ \alpha_1(\cdot) = \left(\frac{\beta_2(\cdot)}{b_{3\ell}} + 2\beta_{2\ell}(\cdot)b_{3\ell}\right)^2 \max\left\{(\eta_n^{\max} - 1)^2, (1 - \eta_n^{\min})^2\right\}, \\ \alpha_2(\cdot) = 2\displaystyle\sum_{i=3}^{n+1}\left(\prod_{\kappa=3}^{i} b_{\kappa\ell}^2 \left(\dfrac{\beta_2(\cdot)}{b_{2\ell}} + 2\beta_i(\cdot)b_{2\ell}\right)^2\right. \\ \qquad \left.\max\left\{\left(\eta_{n-i+2}^{\max} - \eta_{n-i+3}^{\min}\right)^2, \left(\eta_{n-i+3}^{\max} - \eta_{n-i+2}^{\min}\right)^2\right\}\right), \\ \alpha_3(\cdot) = 8\sum_{i=3}^{n+1}\left(\prod_{\kappa=2}^{i} b_{\kappa\ell}^2 \left(\beta_i(\cdot) - \beta_{i+1}(\cdot)\right)^2 \max\left\{\left(\eta_{n-i+2}^{\max} - 1\right)^2, \left(1 - \eta_{n-i+2}^{\min}\right)^2\right\}\right), \end{cases}$$

where $\eta_{n+1}^{\max} = \eta_{n+1}^{\min} = 1$, $0 < \eta_i^{\min} \leqslant 1$, and $1 \leqslant \eta_i^{\max} < +\infty$, $i = 1, 2, \ldots, n$. $\beta_i(b_{i+1,\,\ell}, \ldots, b_{n+1,\,\ell})$, $i = 1, \ldots, n+1$ satisfy $\beta_{n+1}(\cdot) = 1$ and $\beta_{n+2}(\cdot) = 0$. Then, the observer gains κ_i, $i = 1, \ldots, n+1$ are chosen such that

$$\begin{cases} \kappa_{n+1} = -b_{2\ell}\frac{a_{1\ell}}{a_{2\ell}} - \frac{a_{1\ell}}{2a_{2\ell}b_{2\ell}} - \alpha_0\kappa_0, \\ \kappa_{n-i+2} = \frac{a_{1\ell}}{a_{i\ell}}\left(\frac{a_{i-1,\,\ell}}{a_{1\ell}}b_{i\ell}\kappa_{n-i+3} + \frac{a_{i-1,\,\ell}}{a_{i\ell}}b_{i\ell}\prod_{\kappa=2}^{i} b_{\kappa\ell} - \frac{a_{i\ell}}{a_{i+1\ell}}\prod_{\kappa=2}^{i} b_{\kappa\ell}\right), \\ \qquad i = 2, \ldots, n+1, \end{cases}$$

where α_0 is a constant.

Define the positive definite matrix $W \in \mathbb{R}^{(n+1)\times(n+1)}$,

$$\begin{cases} W_{1,1} = \alpha_0 \kappa_0, \\ W_{1,i} = W_{i,1}(\eta(t)) = (1-\eta_{n-i+2}(t)) \left(\dfrac{a_{i-1,\ell}}{a_{i\ell}} b_{i\ell} \prod_{\kappa=2}^{i} b_{\kappa\ell} - \dfrac{a_{i\ell}}{a_{i+1,\ell}} \prod_{\kappa=2}^{i} b_{\kappa\ell} \right) \\ \qquad + (\eta_{n-i+2}(t) - \eta_{n-i+3}(t)) \left(\dfrac{a_{1\ell}}{2b_{2\ell}a_{2\ell}} \prod_{\kappa=2}^{i} b_{\kappa\ell} + \dfrac{a_{i-1,\ell}}{a_{i\ell}} b_{i\ell} \prod_{\kappa=2}^{i} b_{\kappa\ell} \right), \\ W_{i,i} = \kappa_0, \\ W_{i,j} = 0,\ i \neq j,\ i = 2, \ldots, n+1,\ j = 2, \ldots, n+1, \end{cases}$$

where $W_{i,j}$ is the element of the ith row and jth column of the matrix W.

Then, the equation $\tilde{A}^T P + P\tilde{A} = -P_0^T W P_0$ holds, where $P = P_0^T P_0$. Since $\eta_i(t)$ are continuous and bounded, one has

$$\tilde{A}^T P + P\tilde{A} \leqslant -\lambda_0 I, \tag{4}$$

where λ_0 is a positive constant.

Lemma 2.4. A continuous nonlinear function $f(x)$ defined on a compact $\Omega_x \in \mathbb{R}^n$ can be approximated by NNs, i. e.,

$$f(x) = \omega^{*T} \varphi(x) + \varepsilon(x),$$

where $x = (x_1, x_2, \ldots, x_p)^T \in \mathbb{R}^p$ is the input vector, $\varepsilon(x)$ is the approximation error which satisfies $|\varepsilon(x)| \leqslant \varepsilon^*$ and ε^* is a positive constant. The ideal weight vector ω^* and the basis function vector $\varphi(x)$ are defined as

$$\omega^* = (\omega_1^*, \omega_2^*, \ldots, \omega_r^*)^T,$$

$$\varphi(x) = (\varphi_1(x), \varphi_2(x), \ldots, \varphi_r(x))^T,$$

where r is the number of the NNs nodes, $\varphi_i(x)$ are chosen as the commonly used Gaussian functions

$$\varphi_i(x) = exp\left(-\frac{(x-b_i)^T (x-b_i)}{\nu_i^2} \right),\ i = 1, 2, \ldots, r,$$

where $\nu_i > 0$ are the centers of the receptive fields, and b_i are the widths of the Gaussian functions. Let

$$\omega^* = arg \min_{\omega \in \Omega_\omega} \left[\sup_{x \in \Omega_x} \left| f(x) - \hat{\omega}^{*T} \varphi(x) \right| \right],$$

where $\hat{\omega}^* = (\hat{\omega}_1^*, \hat{\omega}_2^*, \ldots, \hat{\omega}_r^*)^T$ is the estimate of ω^*, Ω_ω is a compact set.

3. MAIN RESULTS

3.1. The design of adaptive observer in fault-free

Consider the nonlinear system (1) in the fault-free case as follows

$$\begin{cases} \dot{x}_1 = x_2 + f_1\left(\bar{x}_1\right) + d_{\sigma 1}\left(t\right), \\ \vdots \\ \dot{x}_n = u\left(t\right) + f_n\left(\bar{x}_n\right) + d_{\sigma n}\left(t\right), \\ y = x_1 + d_{\sigma 0}\left(t\right). \end{cases} \tag{5}$$

Propose a first-order output filter strategy for the nonlinear system (5). Then,

$$\dot{x}_0 = -\frac{x_0}{\gamma_0} + \frac{y}{\gamma_0}, \tag{6}$$

where $\gamma_0 \geqslant \frac{\|P\|}{\frac{1}{2}+\|P\|}$ denotes a positive design constant, P is defined in Lemma 2.3.

From Lemma 2.4 and the first-order output filter strategy (6), the nonlinear system (1) can be rewritten as the following extended system,

$$\begin{cases} \dot{x}_0 = x_1 - \frac{x_0}{\gamma_0} + \frac{1-\gamma_0}{\gamma_0}x_1 + d_0\left(t\right), \\ \dot{x}_1 = x_2 + \omega_1^{*T}\varphi_1\left(\bar{x}_1\right) + d_1\left(t\right), \\ \vdots \\ \dot{x}_n = u\left(t\right) + \omega_n^{*T}\varphi_n\left(\bar{x}_n\right) + d_n\left(t\right), \\ \bar{y} = x_0\left(t\right), \end{cases} \tag{7}$$

where, for $i = 1,\ldots,n$, $f_i\left(\bar{x}_i\right) = \omega_i^{*T}\varphi_i\left(\bar{x}_i\right) + \varepsilon\left(\bar{x}_i(t)\right)$, ω_i^* are the optimal weight values, $\varepsilon\left(\bar{x}_i(t)\right)$ are the NNs approximation errors, $d_0\left(t\right) = \frac{d_{\sigma 0}(t)}{\gamma_0}$ and $d_i\left(t\right) = d_{\sigma i}\left(t\right) + \varepsilon\left(\bar{x}_i(t)\right)$ satisfy the following assumption, and $\bar{y}$ represents the output of the extended system (7).

Assumption 3.1. $d_i\left(t\right)$ and $\dot{d}_i\left(t\right)$, $i = 0,\ldots,n$ are assumed to be bound by positive real numbers $\bar{d}_i$ and $\bar{\dot{d}}_i$, respectively, i. e.,

$$\left|d_i\left(t\right)\right| \leqslant \bar{d}_i,\ \left|\dot{d}_i\left(t\right)\right| \leqslant \bar{\dot{d}}_i. \tag{8}$$

A neural network-based adaptive non-fragile state observer is designed as,

$$\begin{cases} \dot{\hat{x}}_0 = \hat{x}_1 - \frac{\hat{x}_0}{\gamma_0} + \frac{1-\gamma_0}{\gamma_0}\hat{x}_1 + \hat{d}_0\left(t\right) + l\kappa_0\left(x_0 - \hat{x}_0\right), \\ \dot{\hat{x}}_1 = \hat{x}_2 + \hat{\omega}_1^{*T}\varphi_1\left(\hat{\bar{x}}_1\right) + \hat{d}_1\left(t\right) + l^2(\kappa_1 + \Delta\kappa_1(t))\left(x_0 - \hat{x}_0\right), \\ \vdots \\ \dot{\hat{x}}_n = u\left(t\right) + \hat{\omega}_n^{*T}\varphi_n\left(\hat{\bar{x}}_n\right) + \hat{d}_n\left(t\right) + l^{n+1}(\kappa_n + \Delta\kappa_n(t))\left(x_0 - \hat{x}_0\right), \end{cases} \tag{9}$$

where $\hat{x}_i$ are the observer state variables, κ_i represent the observer gains obtained from Lemma 2.3, $\hat{d}_i\left(t\right)$ are the estimations of $d_i\left(t\right)$, $i = 0,1,\ldots,n$, $\hat{\omega}_j^*$ and $\Delta\kappa_j(t)$, $j =$

$1, \ldots, n$ are the estimations of the optimal weight values and the observer gain additive disturbances, respectively, $l \geqslant 1$ is the high gain.

Consider the following transformations

$$\begin{cases} \kappa_i + \Delta\kappa_i(t) = \kappa_i \left(1 + \frac{\Delta\kappa_i(t)}{\kappa_i}\right), \\ \left(1 + \frac{\Delta\kappa_i(t)}{\kappa_i}\right) = \theta_i(t),\ i = 1,\ 2,\ \ldots,\ n. \end{cases}$$

The observer (9) can be rewritten as

$$\begin{cases} \dot{\hat{x}}_0 = \hat{x}_1 - \frac{\hat{x}_0}{\gamma_0} + \frac{1-\gamma_0}{\gamma_0}\hat{x}_1 + \hat{d}_0(t) + l\kappa_0(x_0 - \hat{x}_0), \\ \dot{\hat{x}}_1 = \hat{x}_2 + \hat{\omega}_1^{*T}\varphi_1\left(\hat{\bar{x}}_1\right) + \hat{d}_1(t) + l^2\kappa_1\theta_1(t)(x_0 - \hat{x}_0), \\ \vdots \\ \dot{\hat{x}}_n = u(t) + \hat{\omega}_n^{*T}\varphi_n\left(\hat{\bar{x}}_n\right) + \hat{d}_n(t) + l^{n+1}\kappa_n\theta_n(t)(x_0 - \hat{x}_0), \end{cases} \tag{10}$$

where $\theta_i(t)$ are observer gain multiplicative disturbances satisfying the conditions $\theta_i^{\min} \leqslant \theta_i(t) \leqslant \theta_i^{\max}$ with positive real numbers $\theta_i^{\min}$ and $\theta_i^{\max}$.

From (6) and (10), the following error system can be obtained,

$$\begin{cases} \dot{e}_0 = e_1 - \frac{e_0}{\gamma_0} + \frac{1-\gamma_0}{\gamma_0}e_1 - l\kappa_0(t)e_0 + \tilde{d}_0(t), \\ \dot{e}_1 = e_2 + \omega_1^{*T}\tilde{\varphi}_1\left(\hat{\bar{x}}_1, \bar{x}_1\right) + \tilde{\omega}_1^T\varphi_1\left(\hat{\bar{x}}_1\right) - l^2\kappa_1\theta_1(t)e_0 + \tilde{d}_1(t), \\ \vdots \\ \dot{e}_n = \omega_n^{*T}\tilde{\varphi}_n\left(\hat{\bar{x}}_n, \bar{x}_n\right) + \tilde{\omega}_n^T\varphi_n\left(\hat{\bar{x}}_n\right) - l^{n+1}\kappa_n\theta_n(t)e_0 + \tilde{d}_n(t), \end{cases} \tag{11}$$

where $e_i(t) = x_i(t) - \hat{x}_i(t)$, $\tilde{d}_i = d_i(t) - \hat{d}_i(t)$, $i = 0,\ 1, \ldots,\ n$, $\tilde{\varphi}_j\left(\hat{\bar{x}}_j, \bar{x}_j\right) = \varphi_j(\bar{x}_j) - \varphi_j\left(\hat{\bar{x}}_j\right)$ and $\tilde{\omega}_j = \omega_j^* - \hat{\omega}_j^*$, $j = 1,\ 2,\ \ldots,\ n$.

The adaptive laws of $\dot{\hat{d}}_i(t)$, $i = 0,\ 1, \ldots,\ n$ are designed as,

$$\dot{\hat{d}}_i(t) = \lambda_{3i}\left(e_0(t) - \lambda_{4i}\hat{d}_i(t)\right), \tag{12}$$

where $\lambda_{3i} > 0$ and $\lambda_{4i} > 0$ are some design parameters.

Consider the following coordinate transformations

$$\epsilon_i(t) = \frac{x_i(t) - \hat{x}_i(t)}{l^i},\ i = 0,\ 1,\ \ldots,\ n. \tag{13}$$

Combining (11) and (13), one has

$$\dot{\epsilon}(t) = lA\epsilon(t) + B + C + D + E + F, \tag{14}$$

where $A = \begin{pmatrix} -\kappa_0 & 1 & \cdots & 0 \\ -\kappa_1\theta_1(t) & 0 & \ddots & 0 \\ \vdots & \vdots & \cdots & 1 \\ -\kappa_n\theta_n(t) & 0 & \cdots & 0 \end{pmatrix}$, $B = \begin{pmatrix} -\frac{\epsilon_0}{\gamma_0} \\ 0 \\ \vdots \\ 0 \end{pmatrix}$, $C = \begin{pmatrix} \frac{1-\gamma_0}{\gamma_0}\epsilon_1 l \\ 0 \\ \vdots \\ 0 \end{pmatrix}$,

$$D = \begin{pmatrix} 0 \\ \frac{1}{l}\omega_1^{*T}\tilde{\varphi}_1\left(\hat{\bar{x}}_1, \bar{x}_1\right) \\ \vdots \\ \frac{1}{l^n}\omega_n^{*T}\tilde{\varphi}_n\left(\hat{\bar{x}}_n, \bar{x}_n\right) \end{pmatrix},\ E = \begin{pmatrix} 0 \\ \frac{1}{l}\tilde{\omega}_1^{T}\varphi_1\left(\hat{\bar{x}}_1\right) \\ \vdots \\ \frac{1}{l^n}\tilde{\omega}_n^{T}\varphi_n\left(\hat{\bar{x}}_n\right) \end{pmatrix},\ \text{and } F = \begin{pmatrix} \tilde{d}_0(t) \\ \frac{\tilde{d}_1(t)}{l} \\ \vdots \\ \frac{\tilde{d}_n(t)}{l^n} \end{pmatrix}.$$

3.2. The design of adaptive controller in fault-free

By utilizing the non-fragile state observer as a foundation, the controller for the nonlinear system is developed through the implementation of backstepping techniques. Define the following coordinate transformations:

$$\begin{cases} z_0 = x_0, \\ z_i = \hat{x}_i - \alpha_i^c,\ i = 1, \dots, n, \end{cases} \tag{15}$$

where z_0 and z_i are the error variables, and α_i^c, $i = 1, \dots, n$ are the first-order filter outputs. The first-order filters are designed as

$$\lambda_i \dot{\alpha}_i^c + \alpha_i^c = \alpha_{i-1},\ \alpha_i^c(0) = \alpha_{i-1}(0),\ i = 1, \dots, n, \tag{16}$$

where α_{i-1} and λ_i are the first-order filter inputs and the designed constants. Define

$$\varUpsilon_i = \alpha_i^c - \alpha_{i-1},\ i = 1, \dots, n, \tag{17}$$

where $\varUpsilon_i$ represent the errors between the inputs and outputs of the first-order filter.

The adaptive laws of $\hat{\omega}_i^*(t)$ are designed as follows

$$\dot{\hat{\omega}}_i^*(t) = \lambda_{1i}\left(\lambda_{2i}\epsilon_0(t)\varphi_i\left(\hat{\bar{x}}_i\right) - \hat{\omega}_i^*(t)\right),\ i = 1, \dots, n, \tag{18}$$

where $\lambda_{1i} > 1$ and λ_{2i} are real constants.

Step 1: Construct the following positive definite function:

$$V_{c0} = \frac{1}{2}z_0^2.$$

In light of (7) and (15), the time derivative of V_{c0} is given as

$$\begin{aligned} \dot{V}_{c0} &= z_0\dot{z}_0 \\ &= z_0\left(-\frac{x_0(t)}{\gamma_0} + \frac{x_1(t)}{\gamma_0} + d_0(t)\right) \\ &= z_0\left(-\frac{x_0(t)}{\gamma_0} + \frac{l}{\gamma_0}\epsilon_1(t) + \frac{1}{\gamma_0}\left(z_1 + \varUpsilon_1 + \alpha_0\right) + \tilde{d}_0(t) + \hat{d}_0(t)\right). \end{aligned} \tag{19}$$

According to the Young's inequality, it follows that

$$\begin{aligned} & z_0\left(\frac{l}{\gamma_0}\epsilon_1(t) + \varUpsilon_1\frac{1}{\gamma_0} + \frac{1}{\gamma_0}z_1 + \tilde{d}_0(t)\right) \\ \leqslant & z_0^2\left(\frac{l^2+1}{\gamma_0^2} + \frac{1}{4\gamma_0^2} + 1\right) + z_1^2 + \frac{\tilde{d}_0^2(t) + \epsilon_1^2 + {\varUpsilon_1}^2}{4}. \end{aligned} \tag{20}$$

The virtual control law α_0 is inferred as

$$\alpha_0 = \left(-\frac{4l^2+5}{4\gamma_0} - \rho_0\gamma_0\right) z_0 - \gamma_0\hat{d}_0\left(t\right), \tag{21}$$

where $\rho_0 > 1$ is a design parameter.

Substituting (20) and (21) into (19) yields,

$$\dot{V}_{c0} \leqslant -\rho_0 z_0^2 + \frac{\epsilon_1^2}{4} + \frac{\Upsilon_1^2}{4} + \frac{\tilde{d}_0^2}{4} + z_1^2. \tag{22}$$

Step j $(2 \leqslant j \leqslant n)$: Construct the following positive definite functions:

$$V_{ci} = \frac{1}{2}{z_i}^2.$$

In light of (9) and (15), the time derivative of V_{ci} are deduced as

$$\begin{aligned}\dot{V}_{ci} &= z_i\dot{z}_i \\ &= z_i\left(z_{i+1} + \Upsilon_{i+1} + \alpha_i + {\hat{\omega}_i^*}^T\varphi_i\left(\hat{\bar{x}}_i\right) + \hat{d}_i\left(t\right) + l^{i+1}\kappa_i\theta_i\left(t\right)\epsilon_0\left(t\right) - \dot{\alpha}_i^c\right).\end{aligned} \tag{23}$$

According to the Young's inequality, it leads to

$$\begin{aligned}&z_i\left(z_{i+1} + \Upsilon_{i+1} + l^{i+1}\kappa_i\theta_i\left(t\right)\epsilon_0\left(t\right)\right) \\ \leqslant&\frac{z_i^2}{4} + z_{i+1}^2 + z_i^2 + \frac{\Upsilon_{i+1}^2}{4} + z_i^2 l^{2i+2}\kappa_i^2\theta_i^{\max 2} + \frac{{\epsilon_0}^2\left(t\right)}{4}.\end{aligned} \tag{24}$$

The virtual control law α_i are derived as

$$\alpha_i = \left(-\rho_i - l^{2i+2}\kappa_i^2\theta_i^{\max 2} - 2\right) z_i - {\hat{\omega}_i^*}^T\varphi_i\left(\hat{\bar{x}}_i\right) - \hat{d}_i\left(t\right) + \dot{\alpha}_i^c. \tag{25}$$

where $\rho_i > 0$, $i = 1, \ldots, n-1$ are some design parameters.

Substituting (24) and (25) into (23) yields,

$$\dot{V}_{ci} \leqslant -\rho_i z_i^2 + z_{i+1}^2 + \frac{\epsilon_0^2\left(t\right)}{4} + \frac{\Upsilon_{i+1}^2}{4} - z_i^2. \tag{26}$$

Step $n+1$: Construct the following positive definite function:

$$V_{cn} = \frac{1}{2}{z_n}^2. \tag{27}$$

In light of (9) and (15), the time derivative of V_{cn} is obtained as

$$\begin{aligned}\dot{V}_{cn} =&z_n\dot{z}_n \\ =&z_n\left(u\left(t\right) + {\hat{\omega}_n^*}^T\varphi_n\left(\hat{\bar{x}}_n\right) + \hat{d}_n\left(t\right) + l^{n+1}\kappa_n\theta_n\left(t\right)\epsilon_0\left(t\right) - \dot{\alpha}_n^c\right).\end{aligned} \tag{28}$$

According to the Young's inequality, one has

$$z_n l^{n+1}\kappa_n\theta_n\left(t\right)\epsilon_0\left(t\right) \leqslant z_n^2 l^{2n+2}\kappa_n^2\theta_i^{\max\ 2} + \frac{{\epsilon_0}^2\left(t\right)}{4}. \tag{29}$$

The controller $u(t)$ is inferred as

$$u(t) = \left(-\rho_n - l^{2n+2}\kappa_n^2\theta_i^{\max\ 2} - 1\right) z_n - \hat{\omega}_n^{*T}\varphi_n\left(\hat{\bar{x}}_n\right) - \hat{d}_n(t) + \dot{\alpha}_n^c, \tag{30}$$

where $\rho_n > 0$ is a design parameter.

Substituting (29) and (30) into (28) yields,

$$\dot{V}_{cn} \leqslant -\rho_n z_n^2 + \frac{\epsilon_0^2(t)}{4} - z_n^2. \tag{31}$$

Construct the following function:

$$V_1 = \sum_{i=0}^{n} \frac{1}{2} z_i^2(t),\ i = 1, 2, \ldots, n.$$

By combining (22), (26) and (31), the time derivative of V_1 is calculated as

$$\dot{V}_1 \leqslant -\tilde{\rho}\left\|z\right\|^2 + \frac{n}{4}\left\|\epsilon(\mathrm{t})\right\|^2 + \sum_{i=1}^{n}\frac{\Upsilon_i^2}{4} + \frac{\tilde{\mathrm{d}}_0^2(\mathrm{t})}{4}, \tag{32}$$

where $z = (z_0, \ldots, z_n)^T \in \mathbb{R}^{n+1}$ and $\tilde{\rho} = \min\{\rho_i\},\ i = 0, 1, \ldots, n$.

3.3. Stability analysis in fault-free and fault detection

Theorem 3.2. Consider the fault-free nonlinear system (5) with output measurement noise satisfying Assumption 3.1. Design the non-fragile nonlinear observer (9), the controller (30), the virtual controllers (21), (25), the adaptive laws of NNs (18), and the disturbance observer (12). Then, all signals with the initial conditions defined on a compact set Ω_α are SGUUB.

Proof. Construct the following Lyapunov function:

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t). \tag{33}$$

where $V_1 = \sum_{i=0}^{n}\frac{1}{2}z_i^2(t)$, $V_2(t) = \epsilon^T P\epsilon$, $V_3 = \sum_{i=1}^{n}\frac{1}{2}\tilde{\omega}_i^T\tilde{\omega}_i$, $V_4 = \sum_{i=0}^{n}\frac{1}{2}\tilde{d}_i^2(t)$, $V_5 = \sum_{i=1}^{n}\frac{1}{2}\Upsilon_i^2$.

According to Lemma 2.3 and the Young!s inequality, the time derivative of $V_2(t)$ can be derived as

$$\begin{aligned}\dot{V}_2(t) =& l\epsilon^T(t)\left(A^TP+PA\right)\epsilon(t)+2\epsilon^T(t)P(B+C+D+E+F)\\ \leqslant& -l\|\epsilon(t)\|^2+2\|\epsilon(t)\|\|P\|(\|B\|+\|C\|+\|D\|+\|E\|+\|F\|)\\ \leqslant& -l\|\epsilon(t)\|^2+\frac{1}{\gamma_0}\|\epsilon(t)\|^2\|P\|^2+\frac{1}{\gamma_0}\|\epsilon(t)\|^2+a_1n^2\bar{\varphi}^2\|\epsilon(t)\|^2\|P\|^2+\frac{1}{a_1}\sum_{i=1}^{n}\tilde{\omega}_i^T\tilde{\omega}_i\\ &+\|\epsilon(t)\|^2\|P\|^2+n^2\bar{\omega}^2+\frac{l}{2}\|\epsilon(t)\|^2+a_2n\|\epsilon(t)\|^2\|P\|^2+\frac{1}{a_2}\sum_{i=0}^{n}\tilde{d}_i^2(t)\\ \leqslant& -\left(\frac{l}{2}-(\frac{1}{\gamma_0}-1)\|P\|^2-\frac{1}{\gamma_0}-a_1n^2\bar{\varphi}^2\|P\|^2-a_2n\|P\|^2\right)\|\epsilon(t)\|^2\\ &+\sum_{i=0}^{n}\frac{1}{a_2}\tilde{d}_i^2(t)+\sum_{i=1}^{n}\frac{1}{a_1}\tilde{\omega}_i^T\tilde{\omega}_i+n^2\bar{\omega}^2,\end{aligned} \tag{34}$$

where a_1 and a_2 are two positive parameters, $\bar{\omega}=\max\{\|\omega_i^*\|\}$, and $\bar{\varphi}=\max\left\{\left\|\varphi_i\left(\hat{\bar{x}}_i\right)\right\|\right\}$.

Similarly, the derivative of V_3 and V_4 can be expressed as

$$\begin{aligned}\dot{V}_3 \leqslant& \sum_{i=1}^{n}\left(\tilde{\omega}_i^T\left(-\lambda_{1i}\lambda_{2i}\epsilon_0(t)\varphi_i\left(\hat{\bar{x}}_i\right)-\lambda_{1i}\tilde{\omega}_i+\lambda_{1i}\omega_i\right)\right)\\ \leqslant& \sum_{i=1}^{n}\left(-(\lambda_{1i}-1)\tilde{\omega}_i^T\tilde{\omega}_i+\frac{\lambda_{1i}^2\lambda_{2i}^2\epsilon_0^2(t)\bar{\varphi}^2}{2}+\frac{\lambda_{1i}^2\bar{\omega}^2}{2}\right).\end{aligned} \tag{35}$$

$$\begin{aligned}\dot{V}_4 =& \sum_{i=0}^{n}\left(\tilde{d}_i(t)\dot{d}_i(t)-\tilde{d}_i(t)\lambda_{3i}\epsilon_0(t)-\tilde{d}_i^2(t)\lambda_{3i}\lambda_{4i}+\tilde{d}_i(t)\lambda_{3i}\lambda_{4i}d_i(t)\right)\\ \leqslant& \sum_{i=0}^{n}\left(-\left(\lambda_{3i}\lambda_{4i}-\frac{3}{2}\right)\tilde{d}_i^2+\frac{\bar{\dot{d}}_i^2}{2}+\frac{\lambda_{3i}^2\epsilon_0^2(t)}{2}+\frac{\lambda_{3i}{}^2\lambda_{4i}^2\bar{d}_i^2}{2}\right).\end{aligned} \tag{36}$$

In right of (16), (17), and the Young!s inequality, the time derivative of V_5 is derived as

$$\begin{aligned}\dot{V}_5 \leqslant& \sum_{i=1}^{n}\left(-\frac{\Upsilon_i^2}{\lambda_i}-\Upsilon_i\dot{\alpha}_{i-1}\right)\\ \leqslant& \sum_{i=1}^{n}\left(-\left(\frac{1}{\lambda_i}-1\right)\Upsilon_i^2-\frac{\dot{\alpha}_{i-1}^2}{4}\right).\end{aligned}$$

Define the set $\Omega_\alpha=\left\{\sum_{i=1}^{n}\frac{1}{2}\Upsilon_i^2+\sum_{i=0}^{n}\frac{1}{2}\tilde{d}_i^2(t)+\sum_{i=1}^{n}\frac{1}{2}\tilde{\omega}_i^T\tilde{\omega}_i+\sum_{i=0}^{n}\frac{1}{2}z_i^2(t)+\epsilon^TP\epsilon \leqslant \Phi_3\right\}$ in $\mathbb{R}^{5n+3}$. For any $\Phi_3>0$, $\left|\dot{\alpha}_{i-1}^2\right|$ have the maximum $\bar{\alpha}_{i-1}^2$ on the compact Ω_α. Then, it follows that

$$\dot{V}_5 \leqslant \sum_{i=1}^{n}\left(-\left(\frac{1}{\lambda_i}-1\right)\Upsilon_i{}^2-\frac{\bar{\alpha}_{i-1}^2}{4}\right). \tag{37}$$

According to (32), and (34) – (37), one has

$$\dot{V}(t) \leqslant -\left(\frac{l}{2} - \frac{1}{\gamma_0} - \left(\frac{1}{\gamma_0} - 1 + a_1 n^2 \bar{\varphi}^2 + a_2 n\right) \|P\|^2 - \frac{(n+1)\lambda_{1i}^2 \lambda_{2i}^2 \bar{\varphi}^2 + \lambda_{3i}^2}{2} - \frac{n}{4}\right) \|\epsilon(t)\|^2 - \sum_{i=1}^{n} \left(\lambda_{1i} - 1 - \frac{1}{a_1}\right) \tilde{\omega}_i^T \tilde{\omega}_i - \sum_{i=0}^{n} \left(\lambda_{3i}\lambda_{4i} - \frac{1}{a_2} - \frac{n+7}{4}\right) \tilde{d}_i^2(t) - \sum_{i=1}^{n} \left(\frac{1}{\lambda_i} - \frac{5}{4}\right) \Upsilon_i^2 + \sum_{i=1}^{n} \left(\frac{\bar{\alpha}_{i-1}^2}{4} + \frac{\lambda_{1i}^2 \bar{\omega}^2}{2}\right) + \sum_{i=0}^{n} \left(\frac{\bar{\dot{d}}_i^2}{2} + \frac{\lambda_{3i}^2 \lambda_{4i}^2 \bar{d}_i^2}{2}\right) + n^2 \bar{\omega}^2$$

The parameters l, γ_0, a_1, a_2, λ_{1i}, λ_{2i}, λ_{3i} and λ_i are required to satisfy

$$\frac{l}{2} - (\frac{1}{\gamma_0} - 1) \|P\|^2 - \frac{1}{\gamma_0} - a_1 n^2 \bar{\varphi}^2 \|P\|^2 - a_2 n \|P\|^2 - \frac{(n+1)\lambda_{1i}^2 \lambda_{2j}^2 \bar{\varphi}^2 + \lambda_{3i}^2}{2} - \frac{n}{4} > 0,$$

$$\lambda_{1j} - 1 - \frac{1}{a_1} > 0,\ \lambda_{1j} - 1 - \frac{1}{a_1} > 0,\ \lambda_{3i}\lambda_{4i} - \frac{1}{a_2} - \frac{n+7}{4} > 0,\ \frac{1}{\lambda_i} - \frac{5}{4} > 0.$$

Let $\Phi_2 = \min\{\frac{l}{2} - (\frac{1}{\gamma_0} - 1) \|P\|^2 - \frac{1}{\gamma_0} - a_1 n^2 \bar{\varphi}^2 \|P\|^2 - a_2 n \|P\|^2 - \frac{(n+1)\lambda_{1i}^2 \lambda_{2j}^2 \bar{\varphi}^2 + \lambda_{3i}^2}{2} - \frac{\lambda_{3i}^2}{2} - \frac{n}{4}, \lambda_{1j} - 1 - \frac{1}{a_1}, \lambda_{3i}\lambda_{4i} - \frac{1}{a_2} - \frac{n+7}{4}, \frac{1}{\lambda_i} - \frac{5}{4}\}$ and $\Phi_3 = \sum_{i=1}^{n} \left(\frac{\bar{\alpha}_{i-1}^2}{4} + \frac{\lambda_1 i^2 \bar{\omega}^2}{2}\right) + \sum_{i=0}^{n} \left(\frac{\bar{\dot{d}}_i^2}{2} + \frac{\lambda_{3i}^2 \lambda_{4i}^2 \bar{d}_i^2}{2}\right) + n^2 \bar{\omega}^2$. Then,

$$\dot{V}(t) \leqslant -\Phi_2 V(t) + \Phi_3, \tag{38}$$

Since Φ_2 is independent of the variable $\bar{\alpha}_{i-1}$ and the inequality $\Phi_3 \geqslant \frac{\Phi_3}{\Phi_2}$ holds by adjusting the selection of design parameters, then, $\frac{\Phi_3}{\Phi_2} + \left(V(0) - \frac{\Phi_3}{\Phi_2}\right) e^{-\Phi_2} \leq \Phi_3$ holds. From (38), we can obtain that

$$0 \leqslant V(t) \leqslant \frac{\Phi_3}{\Phi_2} + \left(V(0) - \frac{\Phi_3}{\Phi_2}\right) e^{-\Phi_2 t}, \tag{39}$$

which means that Ω_α is an invariant set. Moreover, let $d = \frac{V(0)}{m} + \frac{\Phi_3}{\Phi_2} > 0$, where m is a parameter. The following discussion will be conducted in two cases.

Case 1: If $0 < m < 1$, when $t \geqslant 0$, the following inequality holds

$$V(t) \leqslant V(0)e^{-\Phi_2} + \frac{\Phi_3}{\Phi_2} < d.$$

Case 2: If $1 < m < +\infty$, when $t \geqslant \frac{\ln m}{\Phi_2}$, the following inequality holds

$$V(t) \leqslant V(0)e^{-\Phi_2} + \frac{\Phi_3}{\Phi_2} \leqslant d.$$

In summary, it can be deduced that there exist two positive real numbers d and T such that $V(t) \leqslant d$, when $t > T$. Therefore, the signals e_i, z_i, $\hat{d}_i$, $\hat{\omega}_i^*$ and Υ_i are bounded. According to (21), it follows that α_0 is bounded. Since $\Upsilon_1 = \alpha_1^c - \alpha_0$, it can be established that α_1^c is also bounded. From $z_1 = \hat{x}_1 - \alpha_1^c$, it can be obtained that $\hat{x}_1$ is bounded. It implies that x_1 is bounded because $e_1 = x_1 - \hat{x}_1$. Similar reasoning can be used to prove that α_i, $i = 1, \ldots, n$, α_j^c, $\hat{x}_j$, and x_j, $j = 2, \ldots, n$ are also bounded. The proof is completed. □

In order to detect faults, the following evaluation function is introduced [2],

$$P(t) = \sqrt{\frac{1}{T}\int_t^{t+T} {e_0}^T(\tau)\, e_0(\tau)\, d\tau}. \tag{40}$$

Then, the threshold is obtained as

$$P_{th} = \sup_{\beta(t)\equiv 0} P(t).$$

Therefore, the fault can be detected using the following decision logic

$$\begin{cases} P(t) \leqslant P_{th}, \ Fault - free, \\ P(t) > P_{th}, \ Faulty. \end{cases}$$

Remark 3.3. Since only the output is measurable, the FD threshold in this paper depends the filtered output observation error with measurement noise. Compared with the existing results [31], when the system is subject to output measurement noise, we can only obtain the output observation error with measurement noise $e_1 + d_0$, rather than its exact value. Therefore, by constructing an extended system through an output low-pass filter, the output noise is put into the state equation and estimated by a disturbance observer. Then, the extended system output observation error e_0 can be obtained, and applied to construct the evaluation function. Therefore, the impact of measurement noise on fault diagnosis accuracy can also be reduced.

Remark 3.4. Due to the unavoidable presence of truncation errors in numerical calculations or the aging of sensor equipment, gains of an observer may drift [3, 16, 18, 30]. They can lead to fluctuations of the output observation errors, and result in the decrease in accuracy of fault diagnosis. This problem can be effectively solved by designing a non-fragile observer based on a time-varying matrix inequality.

3.4. The design of adaptive observer in faulty

According to the evaluation function (40), after the fault has been detected, the fault-tolerant control strategies are activated. To ensure that all signals of the closed-loop system are SGUUB, we use NNs to estimate the unknown nonlinear terms and the fault term.

Consider the nonlinear system (1) in the faulty case, i. e., $\beta(t - t_x) = 1$,

$$\begin{cases} \dot{x}_1 = x_2 + f_1(\bar{x}_1) + d_{\sigma 1}(t), \\ \vdots \\ \dot{x}_n = u(t) + f_n(\bar{x}_n) + \eta(\bar{x}_n) + d_{\sigma n}(t), \\ y = x_1 + d_{\sigma 0}(t). \end{cases} \tag{41}$$

Similar to the system (7), the system (41) can also be rewritten as,

$$\begin{cases} \dot{x}_0 = x_1 - \frac{x_0}{\gamma_{c0}} + \frac{1-\gamma_{c0}}{\gamma_{c0}} x_1 + d_{c0}(t), \\ \dot{x}_1 = x_2 + \omega_{c1}^{*T} \varphi_1(\bar{x}_1) + d_{c1}(t), \\ \vdots \\ \dot{x}_n = u(t) + \omega_{cn}^{*T} \varphi_n(\bar{x}_n) + \varpi^{*T} \psi(\bar{x}_n) + d_{cn}(t), \\ \bar{y} = x_0, \end{cases} \tag{42}$$

where the unknown nonlinear terms and the fault term are approximated by NNs as $f_i(\bar{x}_i) = \omega_{ci}^{*T} \varphi_i(\bar{x}_i) + \varepsilon_c(\bar{x}_i(t)),\ i = 1, \ldots, n$ and $\eta(\bar{x}_n) = \varpi^{*T} \psi(\bar{x}_n) + \varepsilon_n(\bar{x}_n(t))$, ϖ^* and ω_{ci}^*, $i = 1, \ldots, n$ are the optimal weight values, $\varepsilon_c(\bar{x}_i(t)),\ i = 1, \ldots, n-1$ and $\varepsilon_n(\bar{x}_n(t))$ are the NNs approximation errors. $\gamma_{c0} \geqslant \frac{\|P_\sigma\|}{\frac{1}{2}+\|P_c\|}$ denotes a positive design constant, $d_{c0}(t) = \frac{d_{\sigma 0}(t)}{\gamma_{c0}}$ is the bounded unknown measurement noise in the output, $d_{ci}(t) = d_{\sigma i}(t) + \varepsilon_c(\bar{x}_i(t)),\ i = 1, \ldots, n-1$ and $d_{cn}(t) = d_{\sigma n}(t) + \varepsilon_c(\bar{x}_n(t)) + \varepsilon_n(\bar{x}_n(t))$ are the bounded time-varying system disturbances which satisfy the following assumption.

Assumption 3.5. $d_{ci}(t)$ and $\dot{d}_{ci}(t),\ i = 0, \ldots, n$ are assumed to be bound by positive real numbers $\bar{d}_{ci}$ and $\bar{\dot{d}}_{ci}$, respectively, i. e.,

$$|d_{ci}(t)| \leqslant \bar{d}_{ci},\ \left|\dot{d}_{ci}(t)\right| \leqslant \bar{\dot{d}}_{ci}. \tag{43}$$

Similar to (10), a neural network-based adaptive non-fragile state observer with fault is designed as follows

$$\begin{cases} \dot{\hat{x}}_{c0} = \hat{x}_{c1} - \frac{\hat{x}_{c0}}{\gamma_{c0}} + \frac{1-\gamma_{c0}}{\gamma_{c0}} \hat{x}_{c1} + \hat{d}_{c0}(t) + l_c \kappa_{c0} (x_0 - \hat{x}_{c0}), \\ \dot{\hat{x}}_{c1} = \hat{x}_{c2} + \hat{\omega}_{c1}^{*T} \varphi_{c1}\left(\hat{\bar{x}}_{c1}\right) + \hat{d}_{c1}(t) + l_c^2 \kappa_{c1} \theta_{c1}(t) (x_0 - \hat{x}_{c0}), \\ \vdots \\ \dot{\hat{x}}_{cn} = u(t) + \hat{\omega}_{cn}^{*T} \varphi_{cn}\left(\hat{\bar{x}}_{cn}\right) + \hat{\varpi}^{*T} \psi\left(\hat{\bar{x}}_{cn}\right) + \hat{d}_{cn}(t) + l_c^{n+1} \kappa_{cn} \theta_{cn}(t) (x_0 - \hat{x}_{c0}), \end{cases} \tag{44}$$

where for $i = 0, 1, \ldots, n$, $\hat{x}_{ci}$ are the observer state variables, κ_{ci} represent the observer gains obtained from Lemma 2.3, $\hat{d}_{ci}(t)$ are the estimations of $d_{ci}(t)$. For $i = 1, 2, \ldots, n$, $\hat{\omega}_{ci}^*$ and $\hat{\varpi}^*$ are the estimations of the optimal weight values, and θ_{ci} are disturbances of observer gains satisfying the conditions $\theta_{ci}^{\min} \leqslant \theta_{ci}(t) \leqslant \theta_{ci}^{\max}$, where $\theta_{ci}^{\min}$ and $\theta_{ci}^{\max}$ are positive parameters. According to (42), we are able to establish the following error system

$$\begin{cases} \dot{e}_{c0} = e_{c1} - \frac{e_{c0}}{\gamma_{c0}} + \frac{1-\gamma_{c0}}{\gamma_{c0}} e_{c1} - l_c \kappa_{c0} e_{c0} + \tilde{d}_{c0}(t), \\ \dot{e}_{c1} = e_{c2} + \omega_{c1}^{*T} \tilde{\varphi}_{c1}\left(\hat{\bar{x}}_{c1}, \bar{x}_1\right) + \tilde{\omega}_{c1}^{T} \varphi_{c1}\left(\hat{\bar{x}}_{c1}\right) - l_c^2 \kappa_{c1} \theta_{c1}(t) e_{c0} + \tilde{d}_{c1}(t), \\ \vdots \\ \dot{e}_{cn} = \omega_{cn}^{*T} \tilde{\varphi}_{cn}\left(\hat{\bar{x}}_{cn}, \bar{x}_n\right) + \tilde{\omega}_{cn}^{T} \varphi_{cn}\left(\hat{\bar{x}}_{cn}\right) + \varpi^{*T} \tilde{\psi}\left(\hat{\bar{x}}_{cn}, \bar{x}_n\right) \\ \qquad + \tilde{\varpi}^{*T} \psi\left(\hat{\bar{x}}_{cn}, \bar{x}_n\right) - l_c^{n+1} \kappa_{cn} \theta_{cn}(t) e_{c0} + \tilde{d}_{cn}(t) \end{cases} \tag{45}$$

where $e_{ci}(t) = x_i(t) - \hat{x}_{ci}(t)$, $\tilde{d}_{ci} = d_{ci} - \hat{d}_{ci}(t)$, $i = 0, 1, \ldots, n$, $\tilde{\varphi}_{cj}\left(\hat{\bar{x}}_{cj}, \bar{x}_j\right) = \varphi_{cj}(\bar{x}_j) - \varphi_{cj}\left(\hat{\bar{x}}_{cj}\right)$, $\tilde{\psi}\left(\hat{\bar{x}}_{cj}, \bar{x}_j\right) = \psi(\bar{x}_j) - \psi\left(\hat{\bar{x}}_{cj}\right)$, $\tilde{\omega}_{cj} = \omega_{cj}{}^* - \hat{\omega}_{cj}^*$, $j = 1, 2, \ldots, n$ and $\tilde{\varpi} = \varpi^* - \hat{\varpi}^*$. $l_c \geqslant 1$ is the high gain.

The adaptive laws $\hat{d}_{ci}(t)$, $i = 0, 1, \ldots, n$ are designed as

$$\dot{\hat{d}}_{ci}(t) = \lambda_{3ci}\left(e_{c0}(t) - \lambda_{4ci}\hat{d}_{ci}(t)\right),$$

where $\lambda_{3ci} > 0$ and $\lambda_{4ci} > 0$ are the design parameters.

Consider the coordinate transformations as follows

$$\epsilon_{ci}(t) = \frac{x_i(t) - \hat{x}_{ci}(t)}{l_c^i},\ i = 0, 1, \ldots, n. \tag{46}$$

Integrating (45) and (46), we obtain

$$\dot{\epsilon}_c(t) = l_c A_c \epsilon(t) + B_c + C_c + D_c + E_c + F_c + G_c + H_c,$$

where $A_c = \begin{pmatrix} -\kappa_{c0}(t) & 1 & \cdots & 0 \\ -\kappa_{c1}\theta_{c1}(t) & 0 & \ddots & 0 \\ \vdots & \vdots & \cdots & 1 \\ -\kappa_{cn}\theta_{cn}(t) & 0 & \cdots & 0 \end{pmatrix}$, $B_c = \begin{pmatrix} -\frac{\epsilon_{c0}}{\gamma_{c0}} \\ 0 \\ \vdots \\ 0 \end{pmatrix}$, $C_c = \begin{pmatrix} \frac{1-\gamma_{c0}}{\gamma_{c0}}\epsilon_{c1} l \\ 0 \\ \vdots \\ 0 \end{pmatrix}$,

$D_c = \begin{pmatrix} 0 \\ \frac{1}{l_c}\omega_{c1}^{*T}\tilde{\varphi}_{c1}\left(\hat{\bar{x}}_{c1}, \bar{x}_1\right) \\ \vdots \\ \frac{1}{l_c^n}\omega_{cn}^{*T}\tilde{\varphi}_{cn}\left(\hat{\bar{x}}_{cn}, \bar{x}_n\right) \end{pmatrix}$, $E_c = \begin{pmatrix} 0 \\ \frac{1}{l_c}\tilde{\omega}_{c1}^T\varphi_{c1}\left(\hat{\bar{x}}_{c1}\right) \\ \vdots \\ \frac{1}{l_c{}^n}\tilde{\omega}_{cn}^T\varphi_{cn}\left(\hat{\bar{x}}_{cn}\right) \end{pmatrix}$, $F_c = \begin{pmatrix} \tilde{d}_{c0}(t) \\ \frac{\tilde{d}_{c1}(t)}{l_c} \\ \vdots \\ \frac{\tilde{d}_{cn}(t)}{l_c{}^n} \end{pmatrix}$,

$G_c = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ \frac{1}{l_c{}^n}\tilde{\varpi}^T\psi\left(\hat{\bar{x}}_{cn}\right) \end{pmatrix}$, $H_c = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ \frac{1}{l_c{}^n}\varpi^{*T}\tilde{\psi}\left(\hat{\bar{x}}_{cn}, \bar{x}_n\right) \end{pmatrix}$.

Choose the coordinate transformations as

$$\begin{cases} z_{c0} = x_{c0}, \\ z_{ci} = \hat{x}_{ci} - \alpha_{ci}^c,\ i = 1, \ldots, n. \end{cases}$$

where z_{c0} and z_{ci} are the error variables, and α_{ci}^c, $i = 1, \ldots, n$ are the first-order filter outputs. The first-order filters are designed as follows

$$\lambda_{ci}\dot{\alpha}_{ci}^c + \alpha_{ci}^c = \alpha_{c(i-1)},\ \alpha_{ci}^c(0) = \alpha_{c(i-1)}(0),\ i = 1, \ldots, n,$$

where $\alpha_{c(i-1)}$ and λ_{ci} are the first-order filter inputs and the designed constants. Define

$$\Upsilon_{ci} = \alpha_{ci}^c - \alpha_{c(i-1)},\ i = 1, \ldots, n,$$

where Υ_{ci} represent the errors between the inputs and outputs of the first-order filters.

The adaptive laws of $\dot{\hat{\omega}}_{ci}^{*}(t)$ and $\dot{\hat{\varpi}}^{*}(t)$ are designed as follows

$$\dot{\hat{\omega}}_{ci}^{*}(t) = \lambda_{1ci}\left(\lambda_{2ci}\epsilon_0(t)\varphi_{ci}\left(\hat{\bar{x}}_{ci}\right) - \hat{\omega}_{ci}^{*}(t)\right), \; i = 1, \ldots, n, \tag{47}$$

$$\dot{\hat{\varpi}}^{*}(t) = \chi_1\left(\chi_2\epsilon_0(t)\psi\left(\hat{\bar{x}}_{cn}\right) - \hat{\varpi}^{*}(t)\right), \tag{48}$$

where $\lambda_{1ci} > 1,\ \lambda_{1ci},\ \chi_1$ and χ_2 are the design parameters.

Since the controller design process and stability analysis with faulty control are similar to those with fault-free control, the controller and the virtual controller are given as

$$\alpha_{c0} = \left(-\frac{4l_c^2 + 5}{4\gamma_{c0}^2} - \rho_{c0}\gamma_{c0}\right)z_{c0} - \gamma_{c0}\hat{d}_{c0}(t), \tag{49}$$

$$\alpha_{ci} = -\rho_{ci}z_{ci} - z_{ci}l_c{}^{2i+2}\kappa_{ci}^2\theta_{ci}^{\max 2} - 3z_{ci} - \hat{\omega}_{ci}^{*T}\varphi_{ci}\left(\hat{\bar{x}}_{ci}\right) - \hat{d}_{ci}(t) + \dot{\alpha}_{ci}^{c}, \quad i = 1, \ldots, n-1, \tag{50}$$

$$u_c(t) = \left(-\rho_{cn} - l_c^{2n+2}\kappa_n^2\theta_{ci}^{\max 2} - 1\right)z_{cn} - \hat{\varpi}^{*T}\psi\left(\hat{\bar{x}}_{cn}\right) - \hat{\omega}_{cn}^{*T}\varphi_{cn}\left(\hat{\bar{x}}_{cn}\right) - \hat{d}_{cn}(t) + \dot{\alpha}_{cn}^{c}. \tag{51}$$

where $\rho_{c0} > 1$, $\rho_{ci} > 0$, $i = 1, \ldots, n-1$ and $\rho_{cn} > 0$ are the design parameters.

Theorem 3.6. Consider the nonlinear system (41) with fault and output measurement noise under Assumption 3.5. Design the non-fragile nonlinear observer (44). Based on the evaluation function (40), it can be obtained the time instant when a fault occurs. Then, with the virtual controller (49), (50), and the adaptive laws of NNs and the disturbance observer (47), (48), a fault-tolerant controller (51) can be designed to ensure that all signals of the closed-loop system are SGUUB.

Algorithm 1 Procedure of the design parameters selection

1: Configure the radial basis functions by choosing the node number, the center and the width;
2: Select the observer gains κ_i such that (4) holds;
3: Select the first-order filters parameters γ_i and the output filter parameter γ_0, and construct the extended system (1);
4: Select the design parameters χ_1, χ_2, λ_{3i}, λ_{4i}, λ_{1i}, λ_{2i} for the update laws $\dot{\hat{\varpi}}^{*}, \dot{\hat{d}}_i, \dot{\hat{\omega}}_i^{*}$;
5: Select the design parameters λ_i, and determine the virtual controllers α_i;
6: The threshold P_{th} is calculated by the algorithm in (40);
7: When a fault is diagnosed, reselect the controller parameters for the FTC

The algorithm of corresponding parameters selection is given by **Algorithm 1**.

4. DEMONSTRATIVE EXAMPLE

Consider a one-link robot system as follows:

$$\begin{aligned} T\ddot{p} + R\dot{p} + E\sin(p) &= \tau + \tau_d \\ W\dot{\tau} + Q\tau &= u - K_m\dot{p} \end{aligned} \tag{52}$$

where p, $\dot{p}$ and $\ddot{p}$ are represent the the link position, velocity and acceleration, respectively, τ is the torque generated by the electrical system, τ_d represents the torque disturbance. $T = 1\,kg/m^2$ represents the mechanical inertia. $R = 0.1\,Ntms/rad$ represents the coefficient of viscous friction at the joint. $E = 10$ is a positive constant related to the load mass and gravity coefficient. $W = 1\,H$ represents the armature inductance. $Q = 1\,\Omega$ represents the armature resistance. $K_m = 0.2\,Nm/A$ represents is the back-emf coefficient. u represents the control input for electric torque.

Consider the case where fault exists in the above system. By introducing the variable $x_1 = p$, $x_2 = \dot{p}$ and $x_3 = \tau$, the system (52) can be rewritten as

$$\begin{cases} \dot{x}_1 = x_2 + f_1(x_1) + d_{\sigma 1}(t), \\ \dot{x}_2 = x_3 + f_2(x_1, x_2) + d_{\sigma 2}(t), \\ \dot{x}_3 = u(t) + f_3(x_1, x_2, x_3) + d_{\sigma 3}(t) + \beta(t - t_x)\,\eta(x_1, x_2, x_3), \\ y = x_1 + d_{\sigma 0}, \end{cases}$$

where the functions $f_1(x_1) = 0.1\sin(x_1)$, $f_2(x_1, x_2) = -(R/T)x_2 - (E/T)\sin(x_1)$, $f_3(x_1, x_2, x_3) = -(K_m/W)x_2 - (Q/W)x_3 + \sin(x_1x_2x_3)$. $d_{\sigma i}(t)$, $i = 1, 2, 3$ are the bounded time-varying system disturbances, $d_{\sigma 0}$ is a bounded unknown measurement noise in the sensor. y denotes the system output. The fault function is chosen as $\eta(x_1, x_2, x_3) = 5 + 0.3\sin(x_1 + x_2 + x_3)$. The time profile of fault is

$$\beta(t - t_x) = \begin{cases} 0, & \text{if } t < t_x, \\ 1, & \text{if } t \geqslant t_x. \end{cases}$$

Assume that the fault occurs at $t_x = 20\,s$.

Similar to (41), the following extended system can be obtained,

$$\begin{cases} \dot{x}_0 = x_1 - \frac{x_0}{\gamma_0} + \frac{1-\gamma_0}{\gamma_0}x_1 + d_0(t), \\ \dot{x}_1 = x_2 + \omega_1^{*T}\varphi_1(x_1) + d_1(t), \\ \dot{x}_2 = x_3 + \omega_2^{*T}\varphi_2(x_1, x_2) + d_2(t), \\ \dot{x}_3 = u(t) + \omega_3^{*T}\varphi_3(x_1, x_2, x_3) + \beta(t - t_x)\,\varpi^{*T}\psi(x_1, x_2, x_3) + d_3(t), \\ \bar{y} = x_0, \end{cases}$$

where $d_0(t) = 0.5\sin(500t)$, $d_1(t) = \cos(500t)$, $d_2(t) = \sin(450t)$, $d_3 = 0.5\sin(450t)$.

According to (9), the following non-fragile observer is presented,

$$\begin{cases} \dot{\hat{x}}_0 = \hat{x}_1 - \frac{\hat{x}_0}{\gamma_0} + \frac{1-\gamma_0}{\gamma_0}\hat{x}_1 + \hat{d}_0(t) + l\kappa_0(x_0 - \hat{x}_0), \\ \dot{\hat{x}}_1 = \hat{x}_2 + \hat{\omega}_1^{*T}\varphi_1(\hat{x}_1) + \hat{d}_1(t) + l^2\kappa_1\theta_1(t)(x_0 - \hat{x}_0), \\ \dot{\hat{x}}_2 = \hat{x}_3 + \hat{\omega}_2^{*T}\varphi_2(\hat{x}_1, \hat{x}_2) + \hat{d}_2(t) + l^3\kappa_2\theta_2(t)(x_0 - \hat{x}_0), \\ \dot{\hat{x}}_3 = u(t) + \hat{\omega}_3^{*T}\varphi_3(\hat{x}_1, \hat{x}_2, \hat{x}_3) + \beta(t - t_x)\,\hat{\varpi}^{*T}\psi(x_1, x_2, x_3) + \hat{d}_3(t) \\ \qquad + l^4\kappa_3\theta_3(t)(x_0 - \hat{x}_0), \end{cases}$$

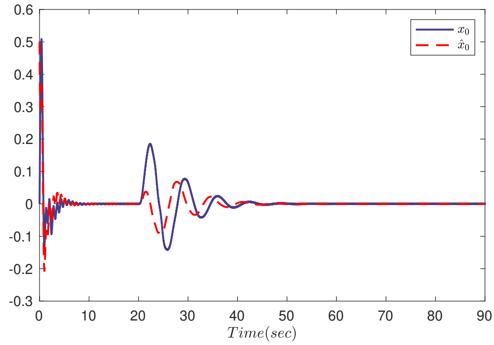

Fig. 1. The trajectories of x_0 and $\hat{x}_0$ under fault-tolerant control.

where the high observer gain $l = 5$, the observer gains $\kappa_0 = 5$, $\kappa_1 = 8$, $\kappa_2 = 10$, $\kappa_3 = 15$, the observer gain disturbances $\theta_1(t) = 1 + 0.1\cos(t)$, $\theta_2(t) = 1 + 0.1\sin(t)$, $\theta_3(t) = 1 + 0.05\cos(t)$ and $\theta_{\max} = 1.1$. Select NN basis functions as $\varphi_1(\hat{x}_1) = exp[-\frac{(\hat{x}_1-5+m)^2}{0.1}]$, $\varphi_2(\hat{x}_1, \hat{x}_2) = exp[-\frac{(\hat{x}_1-5+m)^2+(\hat{x}_2-5+m)^2}{0.1}]$, $\varphi_3(\hat{x}_1, \hat{x}_2, \hat{x}_3) = exp[-\frac{(\hat{x}_1-5+m)^2+(\hat{x}_2-5+m)^2}{0.1} - \frac{(\hat{x}_3-5+m)^2}{0.1}]$, $\psi(\hat{x}_1, \hat{x}_2, \hat{x}_3) = exp[-\frac{(\hat{x}_1-5+m)^2+(\hat{x}_2-5+m)^2+(\hat{x}_3-5+m)^2}{10}]$, $m = 1, \ldots, 10$.

The parameters of the adaptive laws are chosen as $\lambda_{11} = \lambda_{12} = \lambda_{13} = 2/3$, $\lambda_{21} = \lambda_{22} = \lambda_{23} = 3$, $\chi_1 = 0.01$ and $\chi_2 = 1000$. The design parameters in virtual controllers α_i, $i = 0, 1, 2$ and $u(t)$ are given as $\rho_0 = 5$, $\rho_1 = 6$, $\rho_2 = 3\,\rho_3 = 2$ and $\gamma_0 = 1$. The parameters of the first-order filters $\lambda_1 = 0.8$, $\lambda_2 = 0.5$ and $\lambda_3 = 0.001$. The initial values are chosen as $(x_0(0), x_1(0), x_2(0), x_3(0), \hat{x}_0(0), \hat{x}_1(0), \hat{x}_2(0), \hat{x}_3(0)) = (0, 2, -0.2, 0.3, 0.5, 0.5, -0.2, 0.3)$.

The main simulation results are presented in Figure $1-12$. Figure $1-4$ describe the trajectories of system states $x(t) = (x(0), x(1), x(2), x(3))^T$ and the observer states $\hat{x}(t) = (\hat{x}(0), \hat{x}(1), \hat{x}(2), \hat{x}(3))^T$ under FTC case. Figure $5-8$ show the trajectories of system states $x(t)$ and the observer states $\hat{x}(t)$ in the absence of FTC case. The fault is detected at around 22s with the threshold $P_{th} = 0.04$ in Figure 9. Figure 10 and Figure 11 show the trajectories of the norm of the NN weight vectors. Figure 12 shows the trajectory of the controller $u(t)$. In order to demonstrate the effectiveness of our proposed methods, Figure $13-15$ show the trajectories of the system states and the observer states obtained by the method in [31] when the system is in the presence of output measurement noise. Obviously, the states of the system are divergent. It is also shown in Figure 16 that the trajectories of the evaluation function and threshold obtained by the method in [31]. The time detected for the fault is 2.5s. Therefore, it is impossible to initiate FTC strategy rightly when the fault occurs. In summary, based on the method proposed in this paper, all the signals are SGUUB, even if the fault exists.

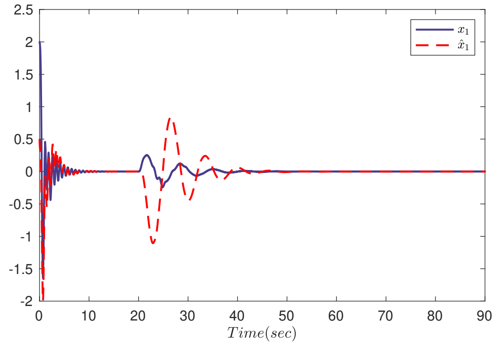

Fig. 2. The trajectories of x_1 and $\hat{x}_1$ under fault-tolerant control.

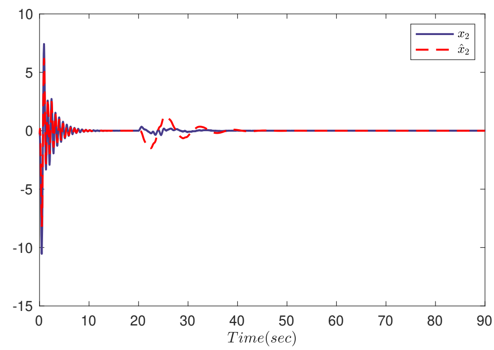

Fig. 3. The trajectories of x_2 and $\hat{x}_2$ under fault-tolerant control.

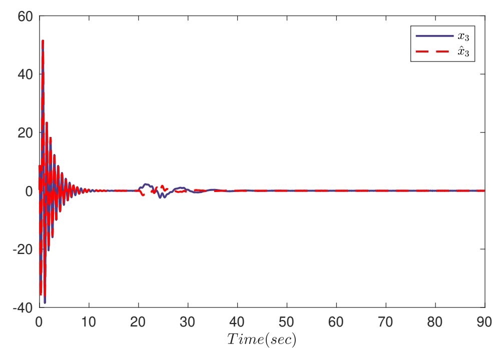

Fig. 4. The trajectories of x_3 and $\hat{x}_3$ under fault-tolerant control.

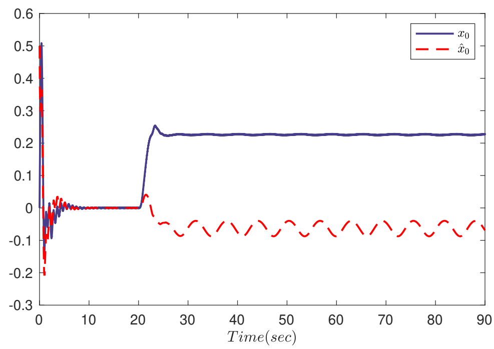

Fig. 5. The trajectories of x_0 and $\hat{x}_0$ without fault-tolerant control.

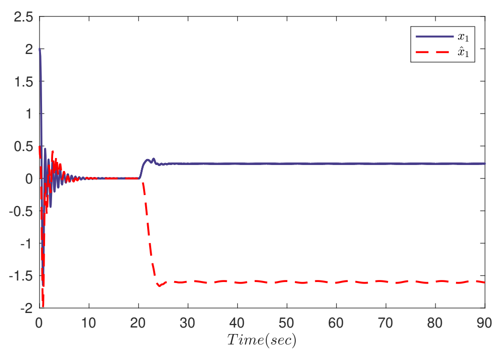

Fig. 6. The trajectories of x_1 and $\hat{x}_1$ without fault-tolerant control.

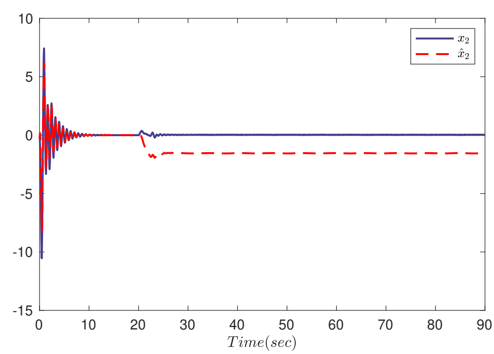

Fig. 7. The trajectories of x_2 and $\hat{x}_2$ without fault-tolerant control.

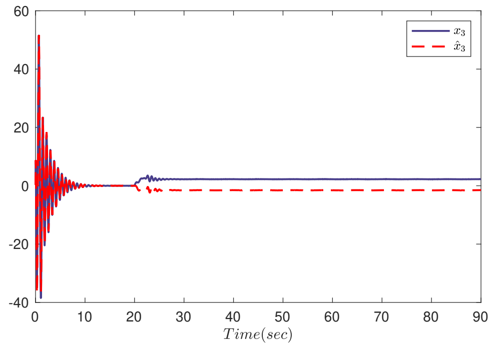

Fig. 8. The trajectories of x_3 and $\hat{x}_3$ without fault-tolerant control.

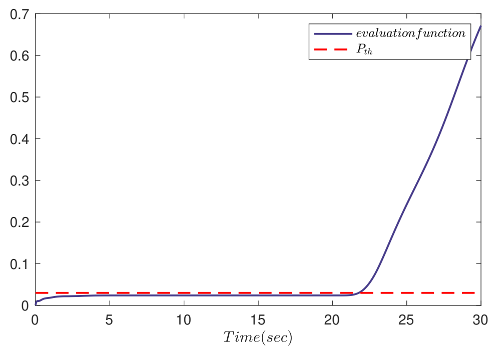

Fig. 9. The trajectories of the evaluation function and threshold.

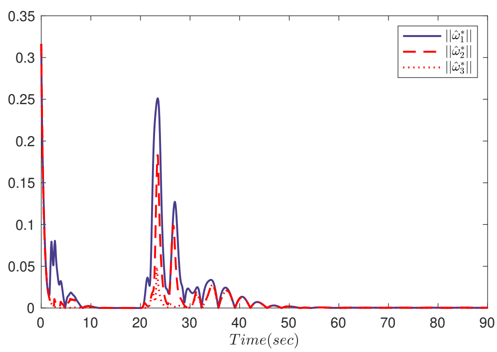

Fig. 10. The trajectories of $\|\hat{\omega}_1^*\|$, $\|\hat{\omega}_2^*\|$, and $\|\hat{\omega}_3^*\|$ under fault-tolerant control.

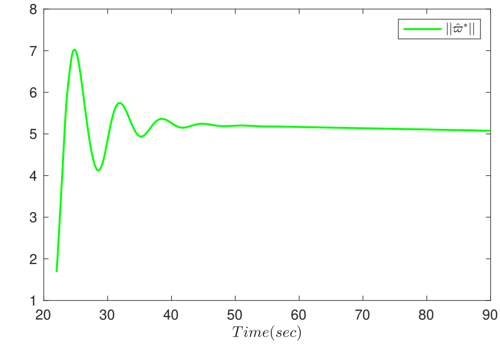

Fig. 11. The trajectory of $\|\varpi^*\|$ under fault-tolerant control.

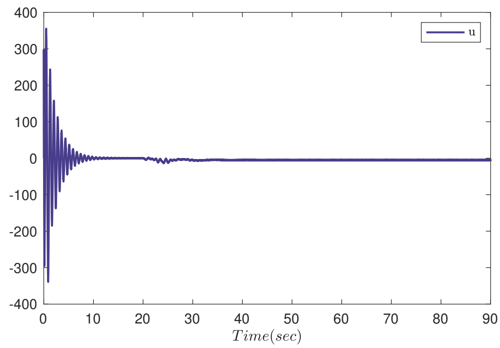

Fig. 12. The trajectory of $u(t)$.

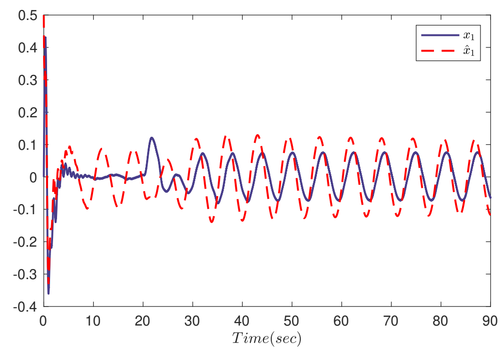

Fig. 13. The trajectories of x_1 and $\hat{x}_1$ obtained by the method in [31].

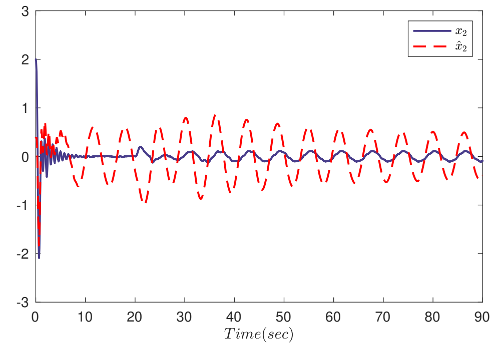

Fig. 14. The trajectories of x_2 and $\hat{x}_2$ obtained by the method in [31].

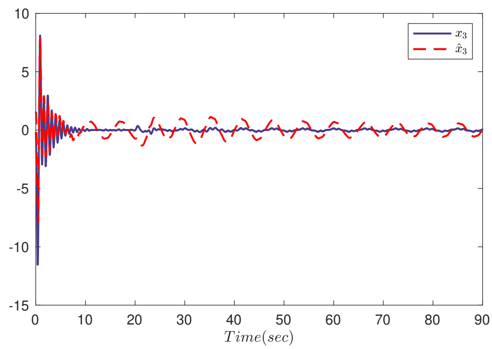

Fig. 15. The trajectories of x_3 and $\hat{x}_3$ obtained by the method in [31].

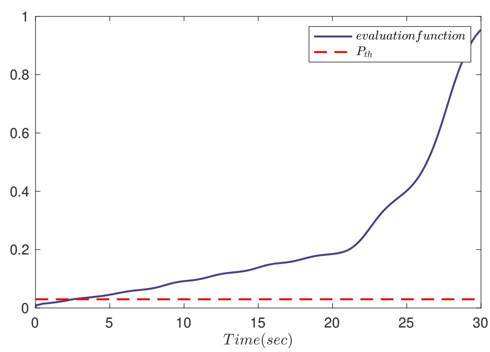

Fig. 16. The trajectories of the evaluation function and threshold obtained by the method in [31].

5. CONCLUSIONS

In this paper, we focused on the active FTC problem for a class of nonlinear systems with measurement noise in the output. Firstly, in order to mitigate the impact of sensor noise and observer gain disturbances on fault diagnosis accuracy, an extended non-fragile observer was established. Then, based on the magnitude of the output observation error, an activation of the FTC scheme was determined. Finally, a neural network adaptive fault-tolerant controller was constructed to ensure that all signals of the closed-loop system are SGUUB. The simulation results showed the effectiveness of our proposed scheme. In the future, we will consider the active FTC problem for a non-strictly nonlinear system.

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