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HOMOLOGICAL DIMENSIONS FOR ENDOMORPHISM ALGEBRAS OF GORENSTEIN PROJECTIVE MODULES

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Abstract. Let A be a CM-finite Artin algebra with a Gorenstein-Auslander generator E, M be a Gorenstein projective A-module and $B = \mathrm{End}_A M$. We give an upper bound for the finitistic dimension of B in terms of homological data of M. Furthermore, if A is n-Gorenstein for $2 \leqslant n < \infty$, then we show the global dimension of B is less than or equal to n plus the B-projective dimension of $\mathrm{Hom}_A(M, E)$. As an application, the global dimension of $\mathrm{End}_A E$ is less than or equal to n.

Keywords: finitistic dimension; Gorenstein projective module; endomorphism algebra

MSC 2020: 16E10, 16G10

1. Introduction

Throughout this paper, let A be an Artin algebra. We denote by A-mod the category of all finitely generated left A-modules. Given an A-module M, we denote by $\mathrm{add}_A M$ the category of all direct summands of finite direct sums of ${}_A M$. The nth syzygy of the A-module M is denoted by $\Omega_A^n(M)$.

Inspired by the work of Auslander and Bridger (see [1]), Enochs and Jenda in [8], [9], [11] introduced the notion of Gorenstein projective modules. In recent years, the class of Gorenstein projective modules has been studied extensively, and is widely used in representation theory and algebraic geometry, see [3], [4], [6], [13], [14].

The finitistic and global dimensions of algebras are important homological invariants and the former one is related to the famous finitistic dimension conjecture. So, it is always an interesting topic to give a characterization of homological dimensions in terms of A and A-modules. Gorenstein projective modules can be viewed as

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a generalization of projective modules. There is much literature on the study of the homological dimensions for endomorphism algebras of projective modules (see for example, [7], [16], [19], [21], [22], [24]), but limited work has been done in the case of Gorenstein projective modules. In this paper we consider endomorphism algebra of the form $B = \mathrm{End}_A M$, where A is a CM-finite Gorenstein algebra, and M is a Gorenstein projective module. While this might seem like a somewhat restricted family of algebras, in fact, every Artin algebra arises in this way since it follows a result of [7] that every Artin algebra is isomorphic to the endomorphism algebra of a projective module over an algebra of finite global dimension.

Following [5], we define the restricted flat dimension of M_B as $\mathrm{rfd}\, M_B = \sup\{i \geqslant 0 : \mathrm{Tor}_i^B(M, X) \neq 0$ for an A-module X with finite projective dimension$\}$. We find that the restricted flat dimension is a useful tool to describe the finitistic dimension for the algebras we consider. Our main results are as follows.

Theorem 1.1. *Let A be a CM-finite Artin algebra with a Gorenstein-Auslander generator E, M be a Gorenstein projective A-module and $B = \mathrm{End}_A M$.*

(1) *If* $\mathrm{fin.dim}\ A \geqslant 4$, *then* $\mathrm{rfd}(M_B) \leqslant \mathrm{fin.dim}\ B \leqslant \mathrm{fin.dim}\ A + \mathrm{rfd}(M_B) + \mathrm{pd}_B \mathrm{Hom}_A(M, E)$.

(2) *If* $\mathrm{fin.dim}\ A \leqslant 3$, *then* $\mathrm{rfd}(M_B) \leqslant \mathrm{fin.dim}\ B \leqslant 3 + \mathrm{rfd}(M_B) + \psi_B(\mathrm{Hom}_A(M, E))$, *where* $\psi_B \colon B\text{-mod} \to \mathbb{N}$ *is the Igusa-Todorov function.*

In [18], Li and Zhang proved that if A is a CM-finite n-Gorenstein algebra with a Gorenstein-Auslander generator E and $2 \leqslant n < \infty$, then the global dimension of $\mathrm{End}_A E$ is less than or equal to n. We extend this result to a more general case.

Theorem 1.2. *Let A be a CM-finite n-Gorenstein algebra with a Gorenstein-Auslander generator E, M be a Gorenstein projective A-module and $B = \mathrm{End}_A M$.*

(1) *If* $2 \leqslant n < \infty$, *then* $\mathrm{gl.dim}\, B \leqslant n + \mathrm{pd}_B \mathrm{Hom}_A(M, E)$.

(2) *If* $n \leqslant 1$, *then* $\mathrm{gl.dim}\, B \leqslant 2 + \mathrm{pd}_B \mathrm{Hom}_A(M, E)$.

This paper is organized as follows. In Section 2, we recall some definitions and basic facts. The proofs of the results will be given in Section 3.

2. Preliminaries

Let A be an Artin algebra. The finitistic dimension of A is defined to be the supremum of projective dimensions of finitely generated modules having finite projective dimension. Let T_A be a right A-module. Let $\mathcal{C}$ be a subcategory of A-mod and $M \in A$-mod. We denote by $\mathcal{C}\text{-dim}({}_A M)$ the minimal integer m such that there is an exact sequence $0 \to T_m \to \ldots \to T_0 \to M \to 0$ with each $T_i \in \mathcal{C}$ and call it the $\mathcal{C}$-dimension of ${}_A M$.

An A-module M is said to be a Gorenstein projective module if there is an exact sequence of projective modules in A-mod,

$$\ldots \to P_1 \to P_0 \to P^0 \to P^1 \to \ldots,$$

such that $M \cong \mathrm{Im}(P_0 \to P^0)$ and the sequence remains exact after applying $\mathrm{Hom}(-, Q)$ for any projective A-module Q.

We say that $M \in A$-mod has Gorenstein-dimension n if there is an exact sequence $0 \to X_n \to \ldots \to X_1 \to X_0 \to M \to 0$ such that all X_i are Gorenstein projective and that n is minimal among the lengths of such exact sequences. In this case we write $\mathrm{Gpd}_A M = n$.

Let $\mathcal{GP}(A)$ be the full subcategory of A-mod consisting of Gorenstein projective modules. An algebra A is called CM-finite if $\mathcal{GP}(A)$ has only finitely many isomorphism classes of indecomposable objects. Evidently, A is CM-finite if and only if there is a Gorenstein projective A-module E such that $\mathcal{GP}(A) = \mathrm{add}_A E$. Such a module E is then called a Gorenstein-Auslander generator of A, see [12]. Note that $\mathcal{GP}(A)$ is a resolving category.

A Gorenstein ring R is a left and right Noetherian ring with $\mathrm{id}_R R < \infty$ and $\mathrm{id} R_R < \infty$; a Gorenstein ring R is n-Gorenstein if $\mathrm{id}_R R \leqslant n$, and in this case $\mathrm{id} R_R \leqslant n$. An algebra A is Gorenstein if it is a Gorenstein ring.

In studying finitistic dimension conjecture, Igusa and Todorov introduced in [17] a powerful function. The following lemma collects some important properties of the Igusa-Todorov function ψ.

Lemma 2.1. *For any Artin algebra A, there is a function ψ which is defined on the objects of A-mod and takes nonnegative integers as values, such that:*

(1) $\psi(M) = \mathrm{pd}_A M$ *if* $\mathrm{pd}_A M < \infty$;

(2) $\psi(X) \leqslant \psi(Y)$ *whenever* $\mathrm{add}_A X \subseteq \mathrm{add}_A Y$. *The equality holds if* $\mathrm{add}_A X = \mathrm{add}_A Y$;

(3) *If* $0 \to X \to Y \to Z \to 0$ *is an exact sequence in* A-mod *with* $\mathrm{pd}_A Z < \infty$, *then* $\mathrm{pd}_A Z \leqslant \psi(X \oplus Y) + 1$.

To prove the main results, we also need several lemmas. The following lemma is proved in [23], Lemma 3.1.

Lemma 2.2. *Let M be a Gorenstein projective A-module and $B = \mathrm{End}_A M$. Then for any $X \in B$-mod, $\Omega^2_B(X) \cong \mathrm{Hom}_A(M, \Omega^2_A(N) \oplus Q)$ for some $N \in A$-mod with $\mathrm{Gpd}_A(M \otimes_B X) = \mathrm{Gpd}({}_A N)$ and some projective A-module ${}_A Q$.*

Lemma 2.3. is well-known in homological algebra.

Lemma 2.3. *Let A be an Artin algebra and let M be an A-module. If there is an exact sequence*

$$0 \to X_s \to \ldots \to X_1 \to X_0 \to M \to 0$$

of A-modules with $\mathrm{pd}_A X_i \leqslant k$ for all i, then $\mathrm{pd}_A M \leqslant s + k$.

In this paper, we follow the standard terminology and notation used in the representation theory of algebras and relative homological algebra, see [2], [10].

3. Homological dimensions for endomorphism algebras of Gorenstein projective modules

Let M be an A-module, where $B = \mathrm{End}_A M$. Then M is also a right B-module. We first give an upper bound for the finitistic dimension of B in terms of that of A, the restricted flat dimension of M_B and the B-projective dimension of $\mathrm{Hom}_A(M, E)$.

Lemma 3.1. $\mathrm{rfd}(M_B) \leqslant \mathrm{fin.dim}\ B$.

P r o o f. This is essentially a result of Lemma 2.1 in [20]. For convenience we repeat the proof. We assume that $\mathrm{fin.dim}\ B = n < \infty$. For any ${}_BX$ with finite projective dimension, we have $\mathrm{pd}_B X \leqslant n$, thus $\mathrm{Tor}^B_{n+1}(M, X) = 0$. By the definition of restricted flat dimension, we can easily deduce $\mathrm{rfd}\, M_B \leqslant n$. □

Theorem 3.2. *Let A be a CM-finite Artin algebra with a Gorenstein-Auslander generator E, M be a Gorenstein projective A-module and $B = \mathrm{End}_A M$.*

(1) *If* $\mathrm{fin.dim}\ A \geqslant 4$, *then* $\mathrm{rfd}(M_B) \leqslant \mathrm{fin.dim}\ B \leqslant \mathrm{fin.dim}\ A + \mathrm{rfd}(M_B) + \mathrm{pd}_B \mathrm{Hom}_A(M, E)$.

(2) *If* $\mathrm{fin.dim}\ A \leqslant 3$, *then* $\mathrm{rfd}(M_B) \leqslant \mathrm{fin.dim}\ B \leqslant 3 + \mathrm{rfd}(M_B) + \psi_B(\mathrm{Hom}_A(M, E))$, *where $\psi_B\colon B\text{-mod} \to \mathbb{N}$ is the Igusa-Todorov function.*

P r o o f. (1) If either $\mathrm{fin.dim}\ A$ or $\mathrm{rfd}(M_B)$ is infinite, then the inequalities are satisfied trivially. So we assume that $\mathrm{fin.dim}\ A = r < \infty$ and $\mathrm{rfd}(M_B) = t < \infty$.

Let ${}_BY \in B\text{-mod}$ with $\mathrm{pd}({}_BY) < \infty$. Denote by Y_i the ith syzygy of Y for each i. Since $\mathrm{rfd}(M_B) = t$, we have $\mathrm{Tor}^B_i(M, Y_t) = \mathrm{Tor}^B_{i+t}(M, Y) = 0$ for $i \geqslant 1$.

Let $0 \to P_m \to \ldots \to P_1 \to P_0 \to Y_t \to 0$ be the projective resolution of Y_t in B-mod. By applying the functor $M \otimes_{B^-}$ to the above exact sequence, we obtain the following exact sequence:

$$0 \to M \otimes_B P_m \to \ldots \to M \otimes_B P_1 \to M \otimes_B P_0 \to M \otimes_B Y_t \to 0. \tag{2.1}$$

Now consider the exact sequence (2.1). Since $M \otimes_B P_i$ $(i = 0, 1, \ldots, m) \in \operatorname{add}_A M$ and the subcategory of Gorenstein projective modules is closed under direct sums and direct summands, we conclude that $M \otimes_B P_i$ are still Gorenstein projective and thus $\operatorname{Gpd}_A(M \otimes_B Y_t) < \infty$. It follows from [21], Lemma 4.4, that $\operatorname{Gpd}_A(M \otimes_B Y_t) \leqslant r$.

By Lemma 2.2, $\Omega_B^2(Y_t) \cong \operatorname{Hom}_A(M, \Omega_A^2(N) \oplus Q)$ for some $N \in A$-mod with $\operatorname{Gpd}_A(M \otimes_B Y_t) = \operatorname{Gpd}_A(N)$ and some projective A-module ${}_AQ$. Since

$$\operatorname{Gpd}_A(M \otimes_B Y_t) \leqslant r,$$

we obtain that

$$\operatorname{Gpd}_A(\Omega_A^2(N)) \leqslant r - 2.$$

According to [15], Theorem 2.6, there exists an exact sequence

$$0 \to Q_{r-2} \to \ldots \to Q_1 \to G_0 \to \Omega_A^2(N) \oplus Q \to 0 \tag{2.2}$$

in A-mod, where G_0 is Gorenstein projective and Q_i is projective for $i = 1, \ldots, r-2$. If M is Gorenstein projective, then $\operatorname{Ext}_A^i(M, L) = 0$ for all $i \geqslant 1$ and all A-modules L with finite $\operatorname{pd}_A L$, see [14], Theorem 2.20. Therefore, by applying the functor $\operatorname{Hom}_A(M, -)$ to exact sequence (2.2), we obtain the following exact sequence: $0 \to \operatorname{Hom}_A(M, Q_{r-2}) \to \ldots \to \operatorname{Hom}_A(M, Q_1) \to \operatorname{Hom}_A(M, G_0) \to \operatorname{Hom}_A(M, \Omega_A^2(N) \oplus Q) \to 0$. Thus, we have

$$\begin{aligned} \operatorname{pd}_B Y &\leqslant \operatorname{pd}\Omega_B^2(Y_t) + t + 2 = \operatorname{pd}_B \operatorname{Hom}_A(M, \Omega_A^2(N) \oplus Q) + t + 2 \\ &\leqslant r + t + \operatorname{pd}_B \operatorname{Hom}_A(M, E), \end{aligned}$$

and the final inequality follows from Lemma 2.3.

(2) When fin.dim $A \leqslant 3$, according to the proof of the above part, we have the exact sequence $0 \to \operatorname{Hom}_A(M, Q_1) \to \operatorname{Hom}_A(M, G_0) \to \operatorname{Hom}_A(M, \Omega_A^2(N) \oplus Q) \to 0$, namely $0 \to \operatorname{Hom}_A(M, Q_1) \to \operatorname{Hom}_A(M, G_0) \to \Omega_B^2(Y_t) \to 0$. Note that the modules in the above exact sequence are left B-modules since M is an A-B-bimodule. Using the Igusa-Todorov function $\psi_B \colon B\text{-mod} \to \mathbb{N}$, we obtain

$$\begin{aligned} \operatorname{pd}_B Y &\leqslant \operatorname{pd}\Omega_B^2(Y_t) + t + 2 \leqslant \psi_B(\operatorname{Hom}_A(M, Q_1) \oplus \operatorname{Hom}_A(M, G_0)) + 1 + t + 2 \\ &\leqslant \psi_B(\operatorname{Hom}_A(M, E)) + t + 3, \end{aligned}$$

and the inequalities follow from Lemma 2.1. Thus, fin.dim $B \leqslant 3 + \operatorname{rfd}(M_B) + \psi_B(\operatorname{Hom}_A(M, E))$. The proof is completed. □

Note that in the case of Theorem 3.2 (2), fin.dim B is finite if and only if $\operatorname{rfd}(M_B)$ is finite since ψ_B takes nonnegative integers as values.

Corollary 3.3. *Let A be a CM-finite Artin algebra with a Gorenstein-Auslander generator E, and $B = \mathrm{End}_A E$.*
(1) *If* $\mathrm{fin.dim}\ A \geqslant 4$, *then* $\mathrm{rfd}(E_B) \leqslant \mathrm{fin.dim}\ B \leqslant \mathrm{fin.dim}\ A + \mathrm{rfd}(E_B)$.
(2) *If* $\mathrm{fin.dim}\ A \leqslant 3$, *then* $\mathrm{rfd}(E_B) \leqslant \mathrm{fin.dim}\ B \leqslant 3 + \mathrm{rfd}(E_B)$.

Theorem 3.4. *Let A be a CM-finite n-Gorenstein algebra with a Gorenstein-Auslander generator E, M be a Gorenstein projective A-module and $B = \mathrm{End}_A M$.*
(1) *If* $2 \leqslant n < \infty$, *then* $\mathrm{gl.dim}\, B \leqslant n + \mathrm{pd}_B \mathrm{Hom}_A(M, E)$.
(2) *If* $n \leqslant 1$, *then* $\mathrm{gl.dim}\, B \leqslant 2 + \mathrm{pd}_B \mathrm{Hom}_A(M, E)$.

P r o o f. (1) Let ${}_B Y \in B$-mod. By Lemma 2.2, $\Omega^2_B(Y) \cong \mathrm{Hom}_A(M, \Omega^2_A(N) \oplus Q)$ for some $N \in A$-mod with $\mathrm{Gpd}_A(M \otimes_B Y) = \mathrm{Gpd}_A(N)$ and some projective A-module ${}_A Q$. Since A is a CM-finite n-Gorenstein algebra, we have $\mathrm{Gpd}_A N \leqslant n$ by [25], Theorem 7.7.7. According to [14], Theorem 2.10, we have an exact sequence:

$$0 \to K_{n-2} \to K_{n-3} \to \ldots \to K_1 \to K_0 \to \Omega^2_A(N) \to 0,$$

where $K_{n-3}, \ldots, K_0$ are Gorenstein projective A-modules. We also have the exact sequence $0 \to \Omega^2_A(N) \to P_1 \to P_0 \to N \to 0$, where P_0, P_1 are projective A-modules. Therefore, we have the following exact sequence:

$$0 \to K_{n-2} \to K_{n-3} \to \ldots \to K_1 \to K_0 \to P_1 \to P_0 \to N \to 0.$$

According to [14], Theorem 2.20, K_{n-2} is Gorenstein projective since $\mathrm{Gpd}_A N \leqslant n$, and thus $\mathrm{Gpd}_A(\Omega^2_A(N)) \leqslant n - 2$.

By [15], Theorem 2.6, there exists an exact sequence

$$0 \to Q_{n-2} \to \ldots \to Q_1 \to G_0 \to \Omega^2_A(N) \oplus Q \to 0 \tag{3.1}$$

in A-mod, where G_0 is Gorenstein projective and Q_i is projective for $i = 1, \ldots, n-2$. Moreover, by applying the functor $\mathrm{Hom}_A(M, -)$ to the exact sequence (3.1), we obtain the following exact sequence: $0 \to \mathrm{Hom}_A(M, Q_{n-2}) \to \ldots \to \mathrm{Hom}_A(M, Q_1) \to \mathrm{Hom}_A(M, G_0) \to \mathrm{Hom}_A(M, \Omega^2_A(N) \oplus Q) \to 0$. Hence,

$$\mathrm{pd}_B Y \leqslant \mathrm{pd}\Omega^2_B(Y) + 2 = \mathrm{pd}_B \mathrm{Hom}_A(M, \Omega^2_A(N) \oplus Q) + 2 \leqslant n + \mathrm{pd}_B \mathrm{Hom}_A(M, E).$$

(2) In the proof of the above part, $\Omega^2_A(N)$ is Gorenstein projective when $n \leqslant 1$, thus $\mathrm{gl.dim}\, B \leqslant 2 + \mathrm{pd}_B \mathrm{Hom}_A(M, E)$. The proof is completed. □

As applications of Theorem 3.4, we can deduce the following results. Note that Corollary 3.5 is also a result of [18].

Corollary 3.5. *Let A be a CM-finite n-Gorenstein algebra with a Gorenstein-Auslander generator E and $2 \leqslant n < \infty$. Then* $\mathrm{gl.dim}\ \mathrm{End}_A E \leqslant n$.

Corollary 3.6. *Let A be a CM-finite 3-Gorenstein algebra, M be a Gorenstein projective A-module and $B = \mathrm{End}_A M$. Then* fin.dim $B < \infty$.

P r o o f. If A is CM-finite, then there exists a Gorenstein projective A-module E such that $\mathcal{GP}(A) = \mathrm{add}_A E$.

Let ${}_BY \in B$-mod with $\mathrm{pd}_B Y < \infty$. By Lemma 2.2,

$$\Omega_B^2(Y) \cong \mathrm{Hom}_A(M, \Omega_A^2(N) \oplus Q)$$

for some $N \in A$-mod with $\mathrm{Gpd}_A(M \otimes_B Y) = \mathrm{Gpd}_A(N)$ and some projective A-module ${}_AQ$. According to the proof of Theorem 3.4, when $n = 3$, we have the exact sequence $0 \to \mathrm{Hom}_A(M, Q_1) \to \mathrm{Hom}_A(M, G_0) \to \mathrm{Hom}_A(M, \Omega_A^2(N) \oplus Q) \to 0$, namely $0 \to \mathrm{Hom}_A(M, Q_1) \to \mathrm{Hom}_A(M, G_0) \to \Omega_B^2(Y) \to 0$. Note that the modules in the above exact sequence are left B-modules since M is an A-B-bimodule. Using the Igusa-Todorov function $\psi_B\colon\ B$-mod $\to \mathbb{N}$, we obtain

$$\begin{aligned} \mathrm{pd}_B Y &\leqslant \mathrm{pd}\Omega_B^2(Y) + 2 \leqslant \psi_B(\mathrm{Hom}_A(M, Q_1) \oplus \mathrm{Hom}_A(M, G_0)) + 1 + 2 \\ &\leqslant \psi_B(\mathrm{Hom}_A(M, E)) + 3, \end{aligned}$$

and the inequalities follow from Lemma 2.1. Consequently, fin.dim $B < \infty$ since $\psi_B(\mathrm{Hom}_A(M, E))$ is finite. The proof is completed. □

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