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ABSOLUTELY SUMMING OPERATORS ON THE BANACH SPACE OF TOTALLY MEASURABLE FUNCTIONS

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Abstract. Let Σ be a σ-algebra of subsets of a set Ω, and X and Y be Banach spaces. Let $B(\Sigma, X)$ stand for the Banach space of all X-valued totally measurable functions on Ω, equipped with the supremum norm. We study absolutely summing operators $T\colon B(\Sigma, X) \to Y$. We characterize absolutely summing operators $T\colon B(\Sigma, X) \to Y$ in terms of their representing operator-valued measures. It is shown that the classes of dominated operators and absolutely summing operators $T\colon B(\Sigma, X) \to Y$ coincide if and only if every bounded linear operator $U\colon X \to Y$ is absolutely summing.

Keywords: space of totally measurable functions; dominated operator; absolutely summing operator; operator-valued measure

MSC 2020: 47B10, 46G10

1. Introduction and preliminaries

The theory of absolutely summing operators originates in the fundamental paper of Grothendieck [12]. It plays an important role in the modern Banach space theory and has applications in harmonic analysis, approximation theory and operator theory, among others (see [1], [4]–[8], [13], [14], [16]–[19]).

Recall that a bounded linear operator $S\colon G \to H$ between Banach spaces G and H is said to be *absolutely summing* if S carries a weakly unconditionally convergent series in G into absolutely convergent series in H, that is, $\sum\limits_{n=1}^{\infty} \|S(z_n)\|_H < \infty$ whenever (z_n) is a sequence in G such that $\sum\limits_{n=1}^{\infty} |z^*(z_n)| < \infty$ for every $z^* \in G^*$.

The following characterization of absolutely summing operators is well known (see [8], pp. 161–162).

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Theorem 1.1. *For a bounded linear operator $S\colon G \to H$, the following statements are equivalent:*

(i) *S is absolutely summing.*

(ii) *S maps an unconditionally convergent series in G into absolutely convergent series in H.*

(iii) *There exists a constant $c > 0$ such that for any finite set $\{z_1, \ldots, z_n\}$ in G,*

$$(*)\qquad \sum_{i=1}^{n} \|S(z_i)\|_H \leqslant c \sup\Big\{\sum_{i=1}^{n} |z^*(z_i)|\colon\ z^* \in G^*,\ \|z^*\|_{G^*} \leqslant 1\Big\}.$$

The least constant c such that the inequality $(*)$ holds is called the *absolutely summing norm* of S and denoted by $\|S\|_{\mathrm{as}}$. Then the linear space $\mathcal{AS}(G,H)$ of all absolutely summing operators $S\colon G \to H$, equipped with the norm $\|\cdot\|_{\mathrm{as}}$, is a Banach space (see [8], p. 162, [17], 2.2.4, Proposition, p. 38).

It is known that every absolutely summing operator $S\colon G \to H$ is Dunford-Pettis and weakly compact (see [1], Corollary 8.2.15).

From now on we assume that $(X, \|\cdot\|_X)$ and $(Y, \|\cdot\|_Y)$ are Banach spaces over the real or complex field $\mathbb{K}$. Let $(X^*, \|\cdot\|_{X^*})$ and $(Y^*, \|\cdot\|_{Y^*})$ stand for their duals and B_{X^*} denote the closed unit ball in X^*. By $\mathcal{L}(X,Y)$ we denote the space of all bounded linear operators $U\colon X \to Y$, equipped with the operator norm $\|\cdot\|$. Then $\mathcal{AS}(X,Y) \subset \mathcal{L}(X,Y)$ and $\|U\| \leqslant \|U\|_{\mathrm{as}}$ for $U \in \mathcal{AS}(X,Y)$.

Let Σ be a σ-algebra of subsets of a nonempty set Ω. Let $\mathbb{1}_A$ stand for the characteristic function of a set $A \in \Sigma$. By $\mathcal{S}(\Sigma, X)$ we denote the space of all X-valued Σ-simple functions $s = \sum_{i=1}^{n} \mathbb{1}_{A_i} \otimes x_i$, where $A_i \in \Sigma$, $x_i \in X$ for $1 \leqslant i \leqslant n$, and $(\mathbb{1}_{A_i} \otimes x_i)(\omega) = \mathbb{1}_{A_i}(\omega) x_i$ for $\omega \in \Omega$. Let $B(\Sigma, X)$ denote the Banach space of all totally Σ-measurable functions $f\colon \Omega \to X$, that is, the space of the uniform limits of all sequences in $\mathcal{S}(\Sigma, X)$, equipped with the supremum norm $\|\cdot\|$ (see [9], Section 6). In particular, let $B(\Sigma) := B(\Sigma, \mathbb{K})$.

Different classes of linear operators $T\colon B(\Sigma, X) \to Y$ have been studied by Dinculeanu [9], Section 9, [10], Section 1.H, Brooks and Lewis [2], [3], Shuchat [20], Swong [22] and Nowak [15].

For a bounded operator $T\colon B(\Sigma, X) \to Y$, let $m\colon \Sigma \to \mathcal{L}(X,Y)$ stand for its *representing measure*, that is,

$$m(A)(x) := T(\mathbb{1}_A \otimes x) \quad \text{for } A \in \Sigma,\ x \in X.$$

Then

$$T(f) = \int_\Omega f(\omega)\,\mathrm{d}m(\omega) \quad \text{for } f \in B(\Sigma, X),$$

where $\int_\Omega f(\omega)\,\mathrm{d}m(\omega)$ denotes the so-called *immediate integral* (see [9], Section 9, [10], Section 1.G). By $|m|(A)$ we denote the variation of m on a set $A \in \Sigma$, that is, $|m|(A) = \sup \sum \|m(A_i)\|$, where the supremum is taken over all finite Σ-partitions (A_i) of A.

Denote by $\mathrm{bva}(\Sigma, X^*)$ the Banach space of all finitely additive measures $\mu\colon \Sigma \to X^*$ of bounded variation, equipped with the norm $\|\mu\| := |\mu|(\Omega)$. The Banach dual $B(\Sigma, X)^*$ of $B(\Sigma, X)$ can be identified with $\mathrm{bva}(\Sigma, X^*)$ through the linear mapping

$$\Phi\colon\ \mathrm{bva}(\Sigma, X^*) \ni \mu \mapsto \Phi_\mu \in B(\Sigma, X)^*,$$

where $\Phi_\mu(f) = \int_\Omega f(\omega)\,\mathrm{d}\mu(\omega)$ for all $f \in B(\Sigma, X)$ and $\|\Phi_\mu\| = |\mu|(\Omega)$ (see [9], Section 9, Corollary 1, p. 147).

In Section 2 we give a characterization of absolutely summing operators $T\colon B(\Sigma, X) \to Y$ in terms of their representing measures. In Section 3 we show that the classes of dominated and absolutely summing operators $T\colon\ B(\Sigma, X) \to Y$ coincide if and only if every bounded linear operator $U\colon\ X \to Y$ is absolutely summing.

2. A characterization of absolutely summing operators on $B(\Sigma, X)$

In this section we characterize absolutely summing operators $T\colon\ B(\Sigma, X) \to Y$ in terms of their representing measures.

Proposition 2.1. *Let $T\colon\ B(\Sigma, X) \to Y$ be an absolutely summing operator. Then $m(A) \in \mathcal{AS}(X, Y)$ for every $A \in \Sigma$ and $|m|_{\mathrm{as}}(\Omega) \leqslant \|T\|_{\mathrm{as}}$, where $|m|_{\mathrm{as}}$ denotes the variation of the measure m with respect to the norm $\|\cdot\|_{\mathrm{as}}$.*

Proof. Assume that the series $\sum\limits_{n=1}^{\infty} x_n$ is weakly unconditionally convergent in X, that is, $\sum\limits_{n=1}^{\infty} |x^*(x_n)| < \infty$ for all $x^* \in X^*$.

Let $A \in \Sigma$. Then the series $\sum\limits_{n=1}^{\infty} \mathbb{1}_A \otimes x_n$ is weakly unconditionally convergent in $B(\Sigma, X)$ because for every $\mu \in \mathrm{bva}(\Sigma, X^*)$, we have $\sum\limits_{n=1}^{\infty} \left|\int_\Omega (\mathbb{1}_A \otimes x_n)(\omega)\,\mathrm{d}\mu\right| = \sum\limits_{n=1}^{\infty} |\mu(A)(x_n)| < \infty$. Since T is absolutely summing, we get

$$\sum_{n=1}^{\infty} \|m(A)(x_n)\|_Y = \sum_{n=1}^{\infty} \|T(\mathbb{1}_A \otimes x_n)\|_Y < \infty,$$

that is, $m(A) \in \mathcal{AS}(X, Y)$.

Now we show that $|m|_{\mathrm{as}}(\Omega) \leqslant \|T\|_{\mathrm{as}}$.

Let $\mathcal{F}(\mathbb{N})$ denote the family of all finite subsets of the set $\mathbb{N}$ of all positive integers. Assume that $(A_j)_{j=1}^n$ is a Σ-partition of Ω. Let $\varepsilon > 0$ be given and $j \in \{1, \ldots, n\}$.

Choose $\sigma_j \in \mathcal{F}(\mathbb{N})$ and $x_{ij} \in X$ for $i \in \sigma_j$ such that

$$\sum_{i\in\sigma_j} |x^*(x_{ij})| \leqslant 1 \quad \text{for every} \quad x^* \in B_{X^*} \tag{2.1}$$

and

$$\sum_{i\in\sigma_j} \|m(A_j)(x_{ij})\|_Y + \frac{\varepsilon}{2^j} \geqslant \|m(A_j)\|_{\mathrm{as}} \tag{2.2}$$

(see [11], Theorem 3.5). Hence using (2.2), we have

$$\begin{aligned}\sum_{j=1}^n \|m(A_j)\|_{\mathrm{as}} &\leqslant \varepsilon + \sum_{j=1}^n \sum_{i\in\sigma_j} \|m(A_j)(x_{ij})\|_Y = \varepsilon + \sum_{j=1}^n \sum_{i\in\sigma_j} \|T(\mathbb{1}_{A_j} \otimes x_{ij})\|_Y \\ &\leqslant \varepsilon + \|T\|_{\mathrm{as}} \sup\Big\{\sum_{j=1}^n \sum_{i\in\sigma_j} \Big|\int_\Omega (\mathbb{1}_{A_j} \otimes x_{ij})(\omega)\,\mathrm{d}\mu(\omega)\Big| : \\ &\qquad \mu \in \mathrm{bva}(\Sigma, X^*),\ |\mu|(\Omega) \leqslant 1\Big\}.\end{aligned} \tag{2.3}$$

Note that by (2.1) for $x^* \in X^*$, $x^* \neq 0$, we have

$$\sum_{i\in\sigma_j} \Big|\frac{x^*}{\|x^*\|_{X^*}}(x_{ij})\Big| \leqslant 1,$$

so $\sum\limits_{i\in\sigma_j} |x^*(x_{ij})| \leqslant \|x^*\|_{X^*}$. Thus for every $\mu \in \mathrm{bva}(\Sigma, X^*)$ with $|\mu|(\Omega) \leqslant 1$, we get

$$\begin{aligned}\sum_{j=1}^n \sum_{i\in\sigma_j} \Big|\int_\Omega (\mathbb{1}_{A_j} \otimes x_{ij})(\omega)\,\mathrm{d}\mu(\omega)\Big| &= \sum_{j=1}^n \sum_{i\in\sigma_j} |\mu(A_j)(x_{ij})| \\ &\leqslant \sum_{j=1}^n \|\mu(A_j)\|_{X^*} \leqslant |\mu|(\Omega) \leqslant 1.\end{aligned} \tag{2.4}$$

Then in view of (2.3) and (2.4) we obtain that $\sum\limits_{j=1}^n \|m(A_j)\|_{\mathrm{as}} \leqslant \varepsilon + \|T\|_{\mathrm{as}}$, and since $\varepsilon > 0$ is arbitrary, we have $\sum\limits_{j=1}^n \|m(A_j)\|_{\mathrm{as}} \leqslant \|T\|_{\mathrm{as}}$. It follows that $|m|_{\mathrm{as}}(\Omega) \leqslant \|T\|_{\mathrm{as}}$, as desired. $\square$

We will need the following lemma (see [21], Lemma 10).

Lemma 2.2. *For a bounded linear operator $T\colon B(\Sigma, X) \to Y$, the following statements are equivalent:*

(i) *T is absolutely summing.*

(ii) *The restriction $T_{\mathcal{S}(\Sigma,X)}\colon \mathcal{S}(\Sigma, X) \to Y$ is an absolutely summing operator.*

In this case, $\|T\|_{\mathrm{as}} = \|T_{\mathcal{S}(\Sigma,X)}\|_{\mathrm{as}}$.

Proposition 2.3. *Let $T\colon B(\Sigma,X)\to Y$ be a bounded linear operator. If $m(A)\in\mathcal{AS}(X,Y)$ for every $A\in\Sigma$ and $|m|_{\rm as}(\Omega)<\infty$, then T is absolutely summing and $\|T\|_{\rm as}\leqslant|m|_{\rm as}(\Omega)$.*

P r o o f. In view of Lemma 2.2 it is enough to show that the operator $T_{\mathcal{S}(\Sigma,X)}\colon\mathcal{S}(\Sigma,X)\to Y$ is absolutely summing and $\|T_{\mathcal{S}(\Sigma,X)}\|_{\rm as}\leqslant|m|_{\rm as}(\Omega)$.

Indeed, assume that $s_j\in\mathcal{S}(\Sigma,X)$ for $j=1,\dots,n$. Note that there exists a Σ-partition $(A_i)_{i=1}^k$ of Ω such that for every $j\in\{1,\dots,n\}$, $s_j=\sum\limits_{i=1}^k\mathbb{1}_{A_i}\otimes x_{ij}$, where $x_{ij}\in X$ for $i=1,\dots,k$. Hence for $j=1,\dots,n$,

$$T(s_j)=\sum_{i=1}^k T(\mathbb{1}_{A_i}\otimes x_{ij})=\sum_{i=1}^k m(A_i)(x_{ij}),$$

and since $m(A_i)\in\mathcal{AS}(X,Y)$, we get

$$\begin{aligned}(2.5)\qquad \sum_{j=1}^n\|T(s_j)\|_Y&\leqslant\sum_{j=1}^n\sum_{i=1}^k\|m(A_i)(x_{ij})\|_Y=\sum_{i=1}^k\Big(\sum_{j=1}^n\|m(A_i)(x_{ij})\|_Y\Big)\\&\leqslant\sum_{i=1}^k\|m(A_i)\|_{\rm as}\sup\Big\{\sum_{j=1}^n|x^*(x_{ij})|\colon\ x^*\in B_{X^*}\Big\}.\end{aligned}$$

For $i\in\{1,\dots,k\}$, pick $\omega_i\in A_i$ and for $x^*\in B_{X^*}$, let us put

$$\Phi_{i,x^*}(s):=x^*(s(\omega_i))\quad\text{for}\quad s\in\mathcal{S}(\Sigma,X).$$

Then $\Phi_{i,x^*}\in\mathcal{S}(\Sigma,X)^*$ with $\|\Phi_{i,x^*}\|\leqslant1$ and $\Phi_{i,x^*}(s_j)=x^*(x_{ij})$ for $j=1,\dots,n$. Thus for $i=1,\dots,k$, we have

$$\sum_{j=1}^n|x^*(x_{ij})|=\sum_{j=1}^n|\Phi_{i,x^*}(s_j)|\leqslant\sup\Big\{\sum_{j=1}^n|\Phi(s_j)|\colon\ \Phi\in\mathcal{S}(\Sigma,X)^*,\|\Phi\|\leqslant1\Big\},$$

and hence

$$\sup\Big\{\sum_{j=1}^n|x^*(x_{ij})|\colon\ x^*\in B_{X^*}\Big\}\leqslant\sup\Big\{\sum_{j=1}^n|\Phi(s_j)|\colon\ \Phi\in\mathcal{S}(\Sigma,X)^*,\|\Phi\|\leqslant1\Big\}.$$

Then using (2.5), we get

$$\begin{aligned}\sum_{j=1}^n\|T(s_j)\|_Y&\leqslant\Big(\sum_{i=1}^k\|m(A_i)\|_{\rm as}\Big)\sup\Big\{\sum_{j=1}^n|\Phi(s_j)|\colon\ \Phi\in\mathcal{S}(\Sigma,X)^*,\|\Phi\|\leqslant1\Big\}\\&\leqslant|m|_{\rm as}(\Omega)\sup\Big\{\sum_{j=1}^n|\Phi(s_j)|\colon\ \Phi\in\mathcal{S}(\Sigma,X)^*,\|\Phi\|\leqslant1\Big\}.\end{aligned}$$

This means that $T_{\mathcal{S}(\Sigma,X)}\colon\ \mathcal{S}(\Sigma,X)\to Y$ is an absolutely summing operator and $\|T_{\mathcal{S}(\Sigma,X)}\|_{\rm as}\leqslant|m|_{\rm as}(\Omega)$. $\square$

As a consequence of Proposition 2.1 and 2.3, we have the following characterization of absolutely summing operators $T\colon B(\Sigma, X) \to Y$.

Corollary 2.4. *Assume that $T\colon B(\Sigma, X) \to Y$ is a bounded linear operator. Then the following statements are equivalent:*

(i) *T is absolutely summing.*

(ii) *$m(A) \in \mathcal{AS}(X, Y)$ for every $A \in \Sigma$ and $|m|_{\mathrm{as}}(\Omega) < \infty$.*

In this case, $\|T\|_{\mathrm{as}} = |m|_{\mathrm{as}}(\Omega)$.

3. Relationship between dominated and absolutely summing operators on $B(\Sigma, X)$

Following [9], Section 9.3, we can distinguish the class of dominated operators $T\colon B(\Sigma, X) \to Y$.

Definition 3.1. A linear operator $T\colon B(\Sigma, X) \to Y$ is said to be *dominated* if there exists a positive finitely additive measure λ on Σ such that

$$\|T(f)\|_Y \leqslant \int_\Omega \|f(\omega)\|_X \, \mathrm{d}\lambda(\omega) \quad \text{for} \quad f \in B(\Sigma, X).$$

Then we say that T is dominated by λ.

We have the following characterization of dominated operators $B(\Sigma, X) \to Y$ (see [9], Section 9.3, p. 150).

Proposition 3.2. *Let $T\colon B(\Sigma, X) \to Y$ be a bounded linear operator. Then the following statements are equivalent:*

(i) *T is dominated.*

(ii) $|m|(\Omega) < \infty$.

In this case, $|m|$ is the least positive finitely additive measure on Σ dominating T.

It is known that a linear operator $T\colon B(\Sigma) \to Y$ is absolutely summing if and only if its representing measure $m\colon \Sigma \to Y$ has a finite variation (see [6], Theorem 1). This means that the classes of absolutely summing operators and dominated operators $T\colon B(\Sigma) \to Y$ coincide.

Note that if $T\colon B(\Sigma, X) \to Y$ is an absolutely summing operator, then T is dominated because $|m|(\Omega) \leqslant |m|_{\mathrm{as}}(\Omega) < \infty$ (see Proposition 2.1). Now we show that every dominated operator $T\colon B(\Sigma, X) \to Y$ is absolutely summing if and only if every bounded linear operator $U\colon X \to Y$ is absolutely summing.

Lemma 3.3. *Assume that $\mathcal{L}(E, F) \subset \mathcal{AS}(E, F)$. Then there exists $a > 0$ such that $\|U\|_{\mathrm{as}} \leqslant a\|U\|$ for all $U \in \mathcal{L}(E, F)$.*

P r o o f. Assume that $U \in \mathcal{L}(E,F)$, $U_n \in \mathcal{L}(E,F)$ for $n = 1, 2, \ldots$, and $\|U_n\| \to 0$ and $\|U_n - U\|_{\mathrm{as}} \to 0$. Then

$$\|U\| \leqslant \|U_n\| + \|U_n - U\| \leqslant \|U_n\| + \|U_n - U\|_{\mathrm{as}} \to 0.$$

Hence $U = 0$ and by the Closed Graph Theorem, the identity operator $I\colon \mathcal{L}(E,F) \to \mathcal{AS}(E,F)$ is bounded, so there exists $a > 0$ such that $\|U\|_{\mathrm{as}} \leqslant a\|U\|$ for $U \in \mathcal{L}(E,F)$. $\square$

Recall that if X is an abstract $\mathcal{L}_1$-space (e.g., $X = L^1$) and Y is a Hilbert space, then $\mathcal{L}(X,Y) \subset \mathcal{AS}(X,Y)$ (see [14], Theorem 4.1, [12], Corollary 1, p. 59).

Proposition 3.4. *For Banach spaces X and Y, the following statements are equivalent:*

(i) $\mathcal{L}(X,Y) \subset \mathcal{AS}(X,Y)$.

(ii) *Every dominated operator $T\colon B(\Sigma, X) \to Y$ is absolutely summing.*

P r o o f. (i) $\Rightarrow$ (ii) Assume that (i) holds and $T\colon B(\Sigma,X) \to Y$ is a dominated operator, that is, $|m|(\Omega) < \infty$ (see Proposition 3.2). Then by Lemma 3.3 there exists $a > 0$ such that $\|m(A)\|_{\mathrm{as}} \leqslant a\|m(A)\|$ for all $A \in \Sigma$. It follows that $|m|_{\mathrm{as}}(\Omega) \leqslant a|m|(\Omega) < \infty$, and by Proposition 2.3, T is absolutely summing.

(ii) $\Rightarrow$ (i) Assume that (ii) holds and (x_n) is a sequence in X such that $\sum\limits_{n=1}^{\infty}|x^*(x_n)| < \infty$ for every $x^* \in X^*$.

For a fixed $\omega_0 \in \Omega$, let $\delta_{\omega_0}(u) := u(\omega_0)$ for $u \in B(\Sigma)$. Then $\delta_{\omega_0} \in B(\Sigma)^*$ and hence there exists a unique measure $\lambda_{\omega_0} \in ba(\Sigma)^+$ such that $\delta_{\omega_0}(u) = \int_\Omega u(\omega)\,\mathrm{d}\lambda_{\omega_0}(\omega)$ for $u \in B(\Sigma)$. In particular, for $f \in B(\Sigma,X)$, we have $\|f(\omega_0)\|_X = \int_\Omega \|f(\omega)\|_X\,\mathrm{d}\lambda_{\omega_0}(\omega)$.

For $U \in \mathcal{L}(X,Y)$, let us put $T_U(f) := U(f(\omega_0))$ for $f \in B(\Sigma,X)$. Then we have

$$\begin{aligned}\|T_U(f)\|_Y &= \|U(f(\omega_0))\|_Y \leqslant \|U\|\,\|f(\omega_0)\|_X \\ &= \|U\| \int_\Omega \|f(\omega)\|_X\,\mathrm{d}\lambda_{\omega_0}(\omega) = \int_\Omega \|f(\omega)\|_X\,\mathrm{d}\lambda(\omega),\end{aligned}$$

where $\lambda = \|U\|\lambda_{\omega_0}$. This means that $T_U\colon B(\Sigma,X) \to Y$ is a dominated operator and hence T_U is absolutely summing.

Let $\Psi(x) := \mathbb{1}_\Omega \otimes x$ for $x \in X$. Then $\Psi\colon X \to B(\Sigma,X)$ is a bounded linear operator and for every $\Phi \in B(\Sigma,X)^*$, we have $\Phi \circ \Psi \in X^*$. Thus $\sum\limits_{n=1}^{\infty} |\Phi(\Psi(x_n))| < \infty$ and it follows that $\sum\limits_{n=1}^{\infty} \|T_U(\Psi(x_n))\|_Y < \infty$. Note that $T_U(\Psi(x_n)) = U(\Psi(x_n)(\omega_0)) = U(x_n)$. This means that $U \in \mathcal{AS}(X,Y)$, as desired. $\square$

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