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GEOMETRY IN POLAND IN XV–XVIII CENTURIES

WITOLD WIĘSŁAW

The knowledge of geometry in the Middle Ages was rather small. It was restricted only to elementary facts needed for measuring the surfaces of fields and volumes. University at Cracow was founded in 1364. Elements of practical geometry, beginning six books of EUCLID's *Elements*, perspective and optics as well as the elements of arithmetic and music, were lectured at Stobner Collegium at beginnings of the university.

1 Martinus Rex (1422–1460?) and his Geometry [1]

MARCIN FROM ŻURAWICA, called MARTINUS POLONUS (also: MAR-TINUS REX DE PREMISLIA) was one of the first professors at Cracow Academy. He was born in 1422 at Żurawica. He started his study at Cracow Academy in 1438, and received very quickly the title *magister artium*, the highest scientific degree at the univerity. He also studied at Prague University and later at Leipzig University, receiving master's degree at both universites. He lectured mathematics, elementary arithmetic and Euclid's geometry at Cracow Academy. The cathedra of astrology at Cracow University was founded by MARTINUS in 1450. In Bologna he obtained also the title *doctor of medicine*. In next years he was episcopal physician at Cracow (so called *Rex in medicinis*).

The manuscript [1] is one of his known texts. The author discusses there fundamental properties of measuring, dividing them into three parts: *Altimetria*, which deels with hights, *Planimetria*, and *Profundimetria* (measuring of depth). He also shows how to measure practically volumes of bottles and barrels, giving numerical examples. MAR-TINUS distinguishes the theory from the practice of measuring.

2 Nicolaus Copernicus (1473–1543) and De Revolutionibus [2]

We will discuss only the mathematical parts of [2]. The history of the manuscript of [2] is not known exactly. The manuscript was in the hands of the family NOSTITZ in the year 1774, as its exlibris shows. Manuscript was in possession of the library of Prague University (Karlova Universita) in the years 1945–1956. In July 1956, the Government of Czechoslovakia presented the manuscript of *De Revolutionibus* to Polish nation. Now the manuscript of [2] is in the possession of the library of Jagelonian University under the number: sygn.10.000 II.

Since the life of COPERNICUS is well described in many sources, I recall only a few facts from his life. He studied mathematical sciences at first at Cracow in the years 1491–1494, next in Bologna, Padua and Ferrara. In Italy, he studied not only mathematical sciences, but also medicine, Roman and Greek literature, and the law. He lectured mathematics at Rome at 1500. He came back to Poland in 1504.

Mathematical foundations of [2] are contained in Chapters XII–XIV of Book I. Namely, Chapter XII (*De magnitudine rectarum in circulo linearum*) deals with chords of a circle, where similarly as in works of PTOLEMY, COPERNICUS calculates lengths of chords in a circle, corresponding to a given angle, presenting the calculations in tables. Nowadays we have suitable tables of the values of trigonometric functions. Chapter XIII (*De lateribus et angulis triangulorum planarum rectilinearum*) discusses rectangular triangles, their sides and angles, triangles described on a given circle and the Sine Theorem. Chapter XIV (*De triangulis sphaericis*) presents fundamental properties of spherical triangles, and methods of their solutions. He uses no signs and symbols, with the exception of signs of segments of lines. Theorems and their proofs are presented verbally. Let us see as an example Ptolemy Theorem. COPERNICUS formulates it in the following way:

Si quadrilaterum circulo inscriptum fuerit, rectangulum subdiagoniis comprehensum, aequale est eis que sub lateribus oppositis continentur.

It can be translated as:

In quadrilateral inscribed in a circle, rectangle of [its] two diagonals is equal to the sum of rectangles of [its] opposite sides. It means that $AB \times BD = AB \times CD + AD \times BC$, in our contemporary notation.

A proof runs as follows:



Let be the quadrilateral ABCD, inscribed in a circle. I say that rectangle [formed] from AC and BD is equal to [the sum of] rectangles [formed] from AB and CD, and from AD and BC.

We draw an angle ABE equal to the angle CBD. Adding to them EBD we obtain equality of angles ABD and EBC. The angles ACB and BDA are equal, since they both correspond to the arc AB of the circle. Consequently the angles BEC and BAD are also equal.

Since the triangles BCE and BDA have their angles respectively equal, [they are similar triangles], their sides are proportional. It means that BC: BD = EC: AD. Thus rectangle with sides BC and AD equals to the rectangles with sides EC and BD [i.e. $BC \times AD = EC \times BD$]. Since the angles ABE and CBD are equal, by the construction, and angles BAC and BDC are also equal, as corresponding to the arc BC, the triangles ABE and CBD are [also] similar. Thus AB: BD = AE: CD. Consequently rectangle with sides AB and CD equals to the rectangle with sides AE and BD [i.e. $AB \times CD = AE \times BD$]. It implies the theorem: rectangle with sides AC and BD is equal to [the sum of] rectangles with sides AB and CD, and AD and BC, respectively. [Indeed, $AC \times BD = (AC + EC) \times BD = AE \times BD + EC \times BD =$ $AB \times CD + BC \times AD$, since AE + EC = AC.]

It seems that the presented proof is the simplest known proof of the Ptolemy Theorem.

Mathematics can also be found in Chapter III (Book III), where COPERNICUS described the curve obtained by moving of the poles of the Earth, a suitable algebraic curve of degree four. In Chapter VII (Book VI) he applies arithmetic mean to the results of experiments.

He calculated tables of legths of chords: $sub\alpha = 2r \sin \frac{\alpha}{2}$, taking r = 100.000.

Miernicka.

J Linea nie prosta iest/Eró: a nie idžie táť od Puntta do Punttá/ áby sye nie miátá ná strone vniesć.

J Jasie nie pieste Lince, wedny są Os Ergyté/dugié są Játoczóné/dugié Rrzywé: a msjé trem tych/zową Zámotáné.

D'Ergigla Linea iest / Etóza rotoło idac/ pizychodźi do tegóż Punetu/ od Etózégo sye poczetá.

J Zátoczóna iest/która ze wnątrz idzie kolem ná wiérzch.

J Krzywa iest/ Etóza przedśie idźie onosącsycy ná te y ná owe strone.



Stanisław Grzepski: *Geometria* A page from Chapter 3

3 Stanisław Grzepski (1524–1570) [3]

The problem of correct mensuration of fields was so much important in Poland in the XVI century, that it was considered by King's Secretary, STANISŁAW MIŁOSZEWSKI. GRZEPSKI studied at Cracow Academy and also at Wrocław in Jesuits Academy. Many Poles lived then in Königsberg (Królewiec). Grzepski spent there some time, having possibility to get in touch with some persons and ideas of the Reformation. Later, about 1553, he was a teacher at Koźminek in the School of Czech Brethern. In 1556, he came back to Cracow to finish his study. He received the bachelor's degree, nineteen years after starting his university study, the lowest university degree, and next, seven years later, he finally obtained master's degree. One of his students, MIŁOSZEWSKI, proposed to him to write a practical guide for geometers (now: geodesists). GRZEPSKI wrote the book [3] in a short time. The book was used in Poland for about a century. It is the first text from geometry written in Polish language. He introduced there new Polish mathematical terminology. The first part of the book contains fundamental notions of plane geometry, including only definitions, terminology, often either Greek or Latin, and fundamental theorems, but without proofs.

GRZEPSKI presents an approximate squaring the circle on enclosed pages. He draws two perpendicular diameters in a circle. Next he divides each of them into eight equal parts and takes one more part in each direction of the both diameters. In this way he obtains edges of a square, which is approximately equal to the circle. He presents calculations in the case when diameter = 10 ells, and obtains the number 50 square ells. He states that the obtained result for the surface, attributed by him to ALBRECHT DÜRER, is not so exact, as the result obtained in another way, giving the value $50\frac{2}{7}$ square ells. He states that the second calculation is much more exact, than the first one. It is easy to see that DÜRER's construction gives $\frac{25}{8}$ as the approximation for π , i.e. the Babylonian result, and the second gives ARCHIMEDES' approximation $\frac{22}{7}$ for π .

In the second part of the book, GRZEPSKI introduces practical information about measuring of fields, and elements of *altimetria* and *profundimetria*. Moeover, he described different systems of measures used in the XVI century in Poland.

4 Jan Brożek (1585–1652) alias Johannes Broscius

JAN BROŻEK was Rennaissance magister scientiarum at Cracow Academy. He took position of Astrologus Ordinarius in the years 1614–1629, at the Cathedral of Astrology founded at the university by MARCIN KRÓL FROM ZURAWICA. Earlier, in 1605, he was bachelor at the university and he lectured Arithmetica of J. PEURBACH, astronomy based on the papers of SACRO BOSCO, elementary geometry and many others. His activity was well-known. He fought against Jesuits, and he tried (with a success) to save the independence of Cracow Academy from Jesuits. He saved many manuscripts of COPERNICUS, but probably lost many others. He wrote poems and had many scientific and personal connections with many known people of the epoch. But in the first place he was a mathematician. He wrote Gaeodesia Distantiarvm (1610), Dissertatio de Cometa Astrophili (1619), Arithmetica Integrorvm (1620), a textbook for students, De Numeris Perfectis Disceptationes Duae (1637), to mention only few of his publications. In the last text he discovered the Little Fermat Theorem earlier than PIERRE DE FERMAT. BROSCIUS states there some properties of perfect numbers and gives ten theorems without proofs. For example, he states that

$$2^{2n} \equiv 1 \pmod{3}$$
; $2^{4n} \equiv 1 \pmod{5}$; $2^{10n} \equiv 1 \pmod{11}$;
 $2^{12n} \equiv 1 \pmod{13}$

holds for every positive integer n, to mention only half of them. His interest in geometry was concentrated on the star-shaped polygons and on filling the plane by regular polygons. In [4] he stated that the plane can be paved by regular polygons with the number of sides equal only to 3, 4, and 6. He tried to explain in the paper why the bees prefer the last possibility. Recall here that eight years later JOHANN KEPLER in *Harmonices Mundi* found how to pave the plane with different regular polygons.

5 Minor papers from geometry [5],[6],[8]

The papers like the above (loc. cit.) contain very elementary information for geometers. The two page paper [6] is a thesis obtained for few elementary statements from geometry such as: Quadratum est figura habens quatuor latera aequalia, et angulis totidem aequales, seu rectos. [...] (A square has four equal sides and all angles equal each other, i.e. *right angle).* In another place he states that the side and the diagonal of a square are not commensurable.

A much more interesting (but not too much) is the book [8]. Part I: De Lineae Dimensione presents not only different linear measures from the ancient times and their applications, but also some geographical informations. Part II: De Superficierum mensuratione presents fundamental algorithms for measuring plane figures, i.e.: Aream circuli invenire (Measure the surface of a circle), Triangulo dato aequale parallelogrammum constituere (Construct a parallelogram equal to a given triangle [with equal surface]), De Coni dimensione (Measure a cone) etc.

Papers of above type appeared often, but they are not too much important.

6 Stanisław Solski and his monumental work [7]

STANISŁAW SOLSKI was born in 1623. He studied at Jesuit Schools. After some time he taught the pronunciation and mathematics. He was sent to Constantinopole to be there a preacher of Christian prisoners. After coming back to Poland, he decided to live at Cracow, spending there his last years. He died at the end of the XVII century.

STANISŁAW SOLSKI wrote the following books at Cracow:

- Geometra Polski, to iest nauka rysowania, podziału, przemieniania y rozmierzania Liniy, Angułów, Figur y Brył pełnych. Księga I, 1683, 288 p.; Księga II, 1684, 152 pp.; Księga III, 1686, 204 pp.
- 2. Archytekt Polski, to iest nauka ulżenia wszelkich ciężarów, używania potrzebnych machyn ziemnych i wodnych [...], in three books, but only one of them was printed; Cracow, the year 1690, 200 pp.
- 3. Praxis nova et expeditissima mensurandi geometrice quasvis distantias altitudines et profunditates, etc. Cracoviae A. 1688, 136 pp.
- 4. Machina exhibendo motui perpetuo artificiali idonea, mathematicis ad examinandum et perficiendum proposita, A. 1663. Cracoviae. ex offic. Caesarii, 68 pp.
- Nauka o częstem używaniu Nayświętszego Sakramentu i sto sposobów czczenia P. Jezusa. W Krakowie u Szedlów r. 1671, 208 pp.
- 6. Rozmyślania codzienne na cały rok, w niedziele i święta uroczyste z ich własnych ewangeliy a w dni powszedne z ewangeliy niedzielnych przysposobione tygodniowym obrotem etc. w pięŚ części



Title page.

rozłożone. W Krakowie, u Fr. Cezarego, Cz. I 1667, 415 pp.; Cz. II 1677, 462 pp.; Cz. III 1677, pp. 463–918; Cz. V 1681, pp. 919–1354: Cz. V (not known).

Archytekt Polski is the first text from the mechanics written in Polish language. Praxis nova is the Latin version of selected parts of *Geome*try. Book 4 deals with the *perpetuum mobile*. Books 5 and 6 are from Catholic teology. I shall not discuss them here.

Geometra Polski is divided into three Books, each of which contains chapters called Amusements [Zabawy]. Theorems are called Properties. Proofs, called Demonstrations, are rather seldom. At the beginning he states, that the proofs of obvious facts are not necessary. However, there are often references to EUCLID, ARCHIMEDES, APOLLONIUS. Each book of Geometra polski contains Polish and Latin terminology with short explanations. Geometrical terms, definitions and theorems are put together with very practical remarks. The book has chapters dealing with astronomy, cartography, geodesy and geography. He describes many practical instruments. There are also many tables of measure systems, money systems in the neighbour countries etc. In Books II and III there are short verses describing some fundamental algorithms, e.g. an algorithm for calculating square roots.

Contents of Geometra Polski:

- **Amusement I:** Part I gives fundamental geometrical terminology; Part II presents definitions; Part III contains sentences not needing any proofs, which are clear for every clever man, as he writes in other place.
- Amusement II teaches drawing i dividing any lines.
- Amusement III gives a science about angles.
- Amusement IV draws any plane figures.
- Amusement V transforms plane figures into another ones.
- **Amusement VI** gives properties of lines, and figures, as well plane as space figures.
- **Amusement VII** teaches how to measure any lines which are either long, high or deep.

Amusement VIII measures perimeter of any plane figure.



Stanisław Solski: A page with "Amusements"

Amusement IX finds a measure [surface] of any plane figure [...]

Amusement X brings buildings, fortresses etc. onto maps.

Amusement XI divides any field and plane figures into any parts [...]

Amusement XII gives methods of measuring any space figures.

Amusement XIII builds sundials.

Amusement XIV recalls arithmetical rules and forgotten by the reader methods of calculations.

Below we present an approximate squaring the circle taken from [7].

I) Construct a square equal to a given circle.

Solski proceeds as follows. He divides the diameter d = EC into equal 14 parts a, i.e. d = 14a. Let TC = 3a. He draws TL orthogonal to the diameter EC. He claims that the square with the side EL equals to the circle. Then in the right triangle ELT:

$$\begin{split} EL^2 &= LT^2 + ET^2 = ET \times TC + ET^2 = ET(TC + ET) = \\ &= ET \times EC = (\frac{11}{14}d)d = \pi d^2, \end{split}$$

implying Archimedes approximation for the ludolfina: $\pi = \frac{22}{7}$.

II) Another exercise from [7]:

Draw a line from a given point D on the side CL in any triangle CPL dividig the triangle into equal parts. Construction is given in the figure. Let H be the middle point of CL. Draw a line HN parallel to DP. Then CDNP and DNL are equal (i.e. have equal areas).



III) An approximate construction of a 7-gon.

Let HTD and HTF be equilateral triangles in a given circle with radius r and centrum H. SOLSKI claims that DL is approximately equal to the side of a 7-gon inscribed in the circle. Indeed, if DL would be such a side, then we would have:

$$DL = 2r\sin\frac{2\pi}{2\times7} = 2r\sin\frac{\pi}{7} \approx 0.8672r.$$

In the construction above, $DL = \frac{\sqrt{3}}{2}r \approx 0.866r$.

IV) Construct a square inscribed in a given triangle.

We leave construction to the reader.



The constructions are not new. However, SOLSKI selected them and presented in an elegant form.

7 Simon Lhuilier (1750–1840) – a mathematician and a teacher

SIMON ANTOINE JEAN LHUILLIER (1750–1840) was a teacher of mathematics in Geneve. In 1777 he obtained a prize in a competition for a textbook from arithmetic, announced by the Society for Elementary Books (Towarzystwo do Ksiąg Elementarnych) in *Nova Acta Eruditorum* MDCCLXXIV (pages 364-384). He came to Poland, where he was a teacher and librarian at prince ADAM CZARTORYSKI in the years 1777– 1788. He won other competitions for textbooks and he wrote for Polish reformed schools the following textbooks: Geometry (in two parts) and *Algebra*. He wrote the textbooks in French. ANDRZEJ GAWROŃSKI translated them into Polish after discussions under meetings of the Society for Elementary Books. Discussions concentrated mainly on language problems: which scientific terminology should be used in translations. Starting from1795 he was professor at university in Geneve. His mathematical results are concerned around geometry and calculus. He used in 1786 for the first time a notion and the sign LIMES, writing it as LIM (*Exposition lmentaire des Principes des Calculs Suprieurs*. l'Academie Royale des Sciences et Belles-Lettres, pur l'Anne 1786. Berlin). One of LHUILIER's most important results is a generalization of Euler's formula for polyhedrons. EULER remarked in 1750 (a letter to CHRISTIAN GOLDBACH from 14th November 1750) that for any polyhedron without "holes" the formula

$$H + S - A = 2$$

holds. (*H* stands for the number of sides of the polyhedron, *S* for the number of its vertices, and *A* for the number of its edges). LHUILLIER remarked in 1812 that Euler's formula holds only for polyhedrons with genus zero. In other cases, when the polyhedron has genus n (i.e. with n "holes"), the formula

$$H + S - A = 2 - 2n$$

holds. The number is today called *Euler's characteristic of the polyhedron*. He had no satisfactory proof, but the remark was very essential for algebraic topology in future. He published many papers in *Annales de Mathematiques pures et appliques* at the beginning of XIX century. He had also results in elementary isoperimetrical problems (see [11]).

Now we describe shortly his *Geometry* [9, 10].

Part I, accepted by the Society for Elementary Books on 30. X. 1780, has thirteen *Chapters* and two *Annexes*. The exposition is in the spirit of EUCLID, exact and precise, but not so formal as in *Elements*. Already at the beginning, writing on isometries, he warns the teaches:

(f) The Teachers should not be afraid to explain geometrical theorems using motions for their beginning pupils. They are far from any Metaphysics. $[\ldots]$

80. Theorem 2. In every Triangle the sum of its three angles is equal to two right angles. The proof is finished by the remark:

(h) The Theorem is very important. Thus pupils would understand it as exactly as it is possible. [...]

Part II, also accepted on 30. X. 1780, contains stereometry, consists of ten chapters:

I. On relative location of Lines and Planes.

II. On solid Angles.

Preparation to the next Chapters.

On square and cubic Roots.

III. On orthogonal Parallelepipeds.

IV. On non orthogonal Parallelepipeds.

V. On Prisms.

VI. On Prisms and Pyramids.

VII. On Cylinders.

VIII. On Cones.

IX. On Spheres.

X. On similar Bodies.

The book (either of its parts) contains no exercises for the pupils. However, it was typical in textbooks on geometry before XIX century.

(Digressio) On the method called in Latin *Methodus exhaustionis*.

LHUILIER illustrates the method by proving very exactly, that two prisms with equal heights and basis with equal measure must have equal volumes.

The number of editions of the Part II was less than of the Part I.

In opinion of the General Visitor after his visit at Piotrków Schools (25-56. VII 1783):

[...] Lhuilier's Geometry is very good, but for children very dark. In many places the book is too precise [...].

The book was difficult also for teachers. JÓZEF CZECH, professor of mathematics at many schools, e.g. at Cracow Academy, decided in this situation to translate EUCLID's *Elements* (Wilno 1807; next edition 1817). However, he only translated eight Books containing pure geometry. Thus till now we have no translation of full EUCLID's *Elements* into Polish.

8 Jan Śniadecki (1756–1830) and his treatise from algebra and geometry [12, 13]

JAN ŚNIADECKI was one of the most famous Polish mathematicians and astronomers in XVIII and XIX centuries. He was very active in preparation of the Polish educational refom starting from 1775, at high schools and at universities. After being retired at Cracow Academy in 1797 and after some visits abroad, he leaved Cracow and go to Vilnius. He was the rector of Alma Mater Vilnensi (1807-1815). After leaving the rector's chair he was retired.

The book [12]–[13] is a university textbook. It contains elements of algebra, mainly algebra of polynomials and systems of linear equations (vol. I) and analytic geometry (vol. II). In particular, he classified curves defined in the plane by algebraical equations of two variables and degree two. He started in his book a similar classification for quadratic surfaces. However, the classification was not finished in the book.

Note: The references below are arranged chronologically.

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