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THE DEVELOPMENT OF NOTATION IN GRAPH THEORY IN DIFFERENT LANGUAGES

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1 Introduction

The last *Novembertagung* in the second millennium was held in Brno in the Czech Republic. The tradition of *Novembertagungen* was started in 1990 as a common Dutch and German activity and was held annually in one of the two countries at the end of October or the beginning of November until 1998. The last two events were held in 1999 in Denmark and in 2000 in the Czech Republic.

There are several reasons why the topic of this paper which belongs to the history of graph theory and combinatorics is particularly suitable for a *Novembertagung* in the Czech Republic.

The papers of Borůvka and Jarník were pioneer papers in algorithmic graph theory 75 years ago. They will, however, not be discussed in this paper.

In the years after 1960 Czechoslovakia became a strong country in combinatorics. In 1963 the first important international conference on graph theory took place in Smolenice in Slovakia. This combinatorial development was heavily interrupted by the political events of 1968 when many combinatorialists emigrated, mainly to Canada and the USA, e.g. Abrham, Chvátal, Hell, Kotzig, and Rosa. Nevertheless, further international conferences were held in Praha in 1974 and 1982. This tradition was continued in Prachatice (1990) and again in Praha (1998) with two big international conferences attended by participants from many countries of the world.

However, it is not only the combinatorial activity itself which is quite popular in the two countries of former Czechoslovakia. The special topic *Mathematics and Culture* of the 11^{th} *Novembertagung* in Brno leads to a discussion of such cultural creations like the Czech graph theory hymn or reports like the one of Chvátal.

From a general point of view the field of combinatorics will celebrate two anniversaries in the year 2001. In 1876 the definition of a configuration was given in a book of THEODOR REYE. In 1926 the zeroth book on graph theory was published by SAINTE-LAGUË [5].

2 Graphs then and now

In the following two books the reader will find further details on the history of graph theory. [1] gives a good survey on the 200 years after EULER with many translations of original papers. [6] is particularly interesting as far the development in Czechoslovakia is concerned.

2.1 The Königsberg bridges

One of the best known problems in graph theory is the problem of the Königsberg bridges which is, by the way, discussed in the Czech graph theory hymn (see below). The solution of Euler of 1736 is often regarded as the first result in graph theory at all.

In the middle of the 19^{th} century physics and chemistry were interesting fields where graphs could be applied. Papers of KIRCHHOFF, CAYLEY, and SYLVESTER lead to a further development of graph theory. After the zeroth book of 1926 (see above) the first book of KÖNIG [4] was published in 1936. The second book on graph theory of Berge in 1958 was soon translated into other languages, e.g. English and Russian. Since around 1960 graph theory is a theory of its own within mathematics and computer science.

2.2 Earlier graphs

There were already graphs before EULER and the bridges of Königsberg. ATHANASIUS KIRCHER, a Jesuit scientist of the 17^{th} century, showed a wonderful bipartite graph $K_{18,18}$ in one of his many books, the *Ars Magna* of 1669. He followed the tradition of the Catalan scientist RAMON LLULL (13^{th} century).

There is a Roman "map", the *Tabula Peutingeriana*, which shows the important streets of the Roman Empire and its neighbouring regions. However, this graph consists of streets (edges), of towns (vertices), and the distances between towns are given. There is neither a common scale in the "map" nor is the shape of continents and islands similar to their real shapes on earth. It is much more a schematical plan like a modern railway network plan than a real map.

Maybe the oldest graphs which can be seen today are Assyrian "trees of life" (8^{th} century BC) in the British Museum in London.

2.3 Graphs of today

Modern graphs occur today in many situations of daily life. Let me just mention the two following ones which I saw during the *Novembertagung* in Brno.

In a journal of the university in Brno *Události na VUT v Brně 9 (1999)* the high velocity academic computer network in the Czech Republic and in Europe is discussed. In the title of the paper the word *síť* occurs for denoting the graph. this means something like network. However, in a drawing the graph is called *Topologie TEN-34 CZ (1998)*. It happens quite often, also in other languages, that the word topology is used for a graph. The word *graph* is quite unknown among the general public, even in the academic world.

The network of the trams and buses in Brno is called *Síť linek MHD*.

The word *graph* which was introduced by SYLVESTER is not so suitable today since most schoolchildren learn in school what a graph is, but this is a different mathematical object, the graph of a function. The following citation is from [2] where the Czech graph theorist CHVÁTAL explained how he got into first contact with graphs in a Soviet bookshop in his home town Plzeň. This report was given as a birthday talk dedicated to CLAUDE BERGE. In 1964 CHVÁTAL found a book on graphs by *Berzh*. It was the Russian translation of the book of BERGE. CHVÁTAL reports as follows.

> *Second, there was the title: TEORIYA GRAFOV, meaning Theory of Graphs. ...* Theory *of graphs? You draw your x axis, you draw your y axis, you plot the graph of your function, and K: BERZH calls that a theory already? I opened the book, ... and found no parabolas, no straight lines, no sinusoids. No x axis. No y axis. There were no graphs in* Theory of Graphs. *There were plenty of illustrations in the text, but most of them were alluring configurations of small circles connected by straight line segments that sometimes had arrows on them.*

2.4 Notation in graph theory

In fact, the graphs of graph theory are defined as follows. There is a set of *vertices*, the "small circles" of Chvátal. Some of them are connected

by an *edge*, the "straight line segments", some are not connected, that's all. In directed graphs, the *arcs* lead from one vertex to another one with "arrows on them".

The meaning of this notation in graph theory is already very well explained in a chemical textbook of 1866 [3].

> *It must be carefully borne in mind that these graphic formulae are intended to represent* neither the shape *of the molecules,* nor the relative position *of the constituent atoms. The lines connecting the different atoms of a compound, and which might with equal propriety be drawn in any other direction, provided they connected together the same elements, serve* only *to show* the definite disposal of the bonds ... *The* lines *connecting the different atoms of a compound are* but crude symbols *of the bonds of union between them; and it is scarcely necessary to remark that* no such material connexions *exist.*

Sylvester intrduced the word *graph* into mathematical language in 1878 in the journal *Nature*.

> *Every invariant and covariant thus becomes expressible by a* graph *precisely identical with a Kekuléan diagram or chemicograph.*

The word *graph* was then introduced into the German language as a foreign word from English by the Danish mathematician JULIUS PETERSEN in his famous paper of 1891 called *Die Theorie der regulären graphs.*

> *Englische Verfasser haben für ähnliche Figuren den Namen* graph *eingeführt; ich werde diesen Namen beibehalten.*

In French speaking countries the development of notation was a bit different. For a long time the word *réseau* (something like net) was used, and not only the word *graphe*.

3 Mathematics, language, and culture

3.1 The graph theory hymn

Graph theory is, as far as I know, the only mathematical discipline which has a hymn of its own. The text was written by a Czech graph theorist,

BOHDAN ZELINKA in Liberec, and the music was composed by a Czech graph theorist, ZDENĚK RYJÁČEK in Plzeň.

The first verse is as follows.

Přes Pregolu sedm mostů stálo,
na svou dobu nebylo to málo,
královečtí radní hrdi byli,
že si tyto mosty postavili.

In the mean time the text has been translated into English, Hungarian, Polish, German, Afrikaans, and Esperanto. In some cases the translation is quite free. The first verse (in English) is as follows.

Seven bridges spanned the River Pregel,
Many more than might have been expected;
Königsberg's wise leaders were delighted
To have built such very splendid structures.

The refrain which states the result of Euler is given here in the two languages of the *Novembertagung*.

Eulerian graphs all have this restriction:
The degree of any point is even.
That's the oldest graph result
That mankind has ever known.

Eulerscher Graph, Dir ist stets zu eigen,
daß sich die Knoten grad-geradig zeigen.
Es hat dieser erste Satz
im Buch der Graphen seinen Platz.

References

[1] N.L. Biggs, K.E. Lloyd, R.J. Wilson, *Graph theory 1736–1936*, Oxford (1976).

[2] V. Chvátal, "In praise of Claude Berge", *Discrete Math.* 165/166 (1997), 3-9.

[3] E. Frankland, *Lecture notes for chemical students*, London (1866).

[4] D. König, *Theorie der endlichen und unendlichen Graphen*, Leipzig (1936).

[5] A. Sainte-Laguë, *Les réseaux (ou graphes)*, Paris (1926).

[6] P. Šišma, Teorie Grafů 1736–1963, Praha (1997).

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