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In: Stanley P. Franklin and Zdeněk Frolík and Václav Koutník (eds.): General Topology and Its Relations to Modern Analysis and Algebra, Proceedings of the Kanpur topological conference, 1968. Academia Publishing House of the Czechoslovak Academy of Sciences, Praha, 1971. pp. [235]--236.

Persistent URL: http://dml.cz/dmlcz/700558

SOME EQUIVALENT FORMULATION OF ULAM'S PROBLEM

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In 1930, S. M. Ulam asked whether there exists a non-zero countably additive measure which takes only the values 0 and 1, which vanishes on single point sets and which is defined on all subsets of the given set. He showed that if the set is countable there is no such measure. We refer to this as Ulam's problem and formulate it in the following terms: There exists *no* non-zero two-valued countably additive measure which vanishes on single point set and which is defined on all subsets of a given set X, whatever be the cardinality of X. Several equivalent formulations of the above statement have come up in later times [2, p. 206–208]. Among these we mention two.

i) Any product of locally convex bornological spaces is bornological. (Recall that a locally convex space is said to be bornological if every bounded linear functional is continuous.)

ii) Any discrete space is realcompact. (Recall that a completely regular space is realcompact if every free maximal ideal in the space of continuous functions is hyper-real, i.e. the quotient field by the maximal ideal is a totally ordered non-archimedian field [1].)

We give below three other equivalent formulations of Ulam's problem. We recall that an ultrafilter Φ on X is said to be free if the intersection of all its members is empty, and fixed otherwise.

Theorem. *The following statements are equivalent for an infinite set X.*

a) *Given any free ultrafilter Φ on X, there exists a real valued function f such that $f(\Phi)$ is a base of a free ultrafilter.*

b) *Given any free ultrafilter Φ on X, there exists a decreasing sequence of sets $\{F_n\}$, $F_n \in \Phi$ with empty intersection.*

c) *Every ultrafilter Φ which is Cauchy for the weak topology defined on X by $\mathscr{F}(X; R)$, the space of all functions from X to R, is also Cauchy for the strong topology on X defined by uniform convergence on the bounded sets (i.e. bounded pointwise) of $\mathscr{F}(X; R)$.*

d) *Ulam's problem.*

References

[1] *L. Gillman* and *M. Jerison:* Rings of continuous functions. D. Van Nostrand, New York, 1960.

[2] *S. Warner:* Inductive limit of normed algebras. Trans. Amer. Math. Soc. *82* (1956), 190—216.

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