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Some weaker forms of compactness

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SOME WEAKER FORMS OF COMPACTNESS

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Compactness occupies a very important place in topology and so do some of its weaker forms. With the exception of paracompactness, the best-known weaker form of compactness is H-closedness. The theory of such spaces was initiated by Alexandroff and Urysohn [2] in 1929. A Hausdorff space is said to be H-closed if its image is closed in every Hausdorff space in which it can be embedded, or equivalently if for every open cover, there exists a finite sub-family, the closures of whose members cover the space. A T_3 H-closed space is compact. Alexandroff and Urysohn posed two problems about H-closed spaces:

(1) Is a space, every closed subspace of which is H-closed, necessarily compact?

(2) May any Hausdorff space be embedded in an H-closed space as a dense subset?

In 1930, A. Tychonoff [39] made a partial attempt to answer the above problems. He showed that a given topological space X may be embedded in an H-closed space X. In 1937, both the problems were solved by M. H. Stone [38]. M. H. Stone obtained these and some other results by algebraic methods. Some of the results of M. H. Stone had been reobtained by Fomin [8], by topological methods, in 1940 and by Obreanu [24] in 1950. In 1942, A. D. Alexandroff [1] gave a different proof for the theorem that any Hausdorff space X can be extended to an H-closed space.

In 1940 and 1947, M. Katětov [17, 18] did some valuable work on H-closed spaces. He showed that a regularly-closed subset of an H-closed space is H-closed and that a continuous image of an H-closed space is H-closed. He also gave the following characterization of such spaces: A Hausdorff space is H-closed iff for every collection $\{G_\alpha\}$ of open sets of the space which has the finite intersection property, $\bigcap\{\overline{G}_\alpha\}$ is non-empty.

In 1941 and 1943, S. V. Fomin [6, 7] improved upon one of the results of M. Katětov. He introduced the concept of θ-continuity, which is a weaker form of continuity as can easily be seen by the following definition.

Definition 1. [7] *A function $f: X \to Y$ is said to be θ-continuous iff for each point $x \in X$ and each neighbourhood V of $f(x)$, there exists a neighbourhood U of x such that $f(\overline{U}) \subset \overline{V}$.*

We find that a θ-continuous image of an H-closed space is H-closed. In 1941, C. Chevalley and O. Frink [5] proved that an arbitrary product of H-closed spaces is H-closed.

H-closed spaces play an important role in the theory of minimal-Hausdorff spaces. In this direction, work has been done by M. Katětov [17, 18], A. Ramanathan [28, 29], F. Obreanu [23, 24] and others. It is now well known that every minimal Hausdorff space is not necessarily compact and a Hausdorff space is minimal Hausdorff iff it is H-closed and semi-regular.

In 1963, S. Iliadis [12] gave the following characterization of H-closed spaces. A Hausdorff space is H-closed iff it is the image of some (zero-dimensional) compact space under a θ-continuous (irreducible) mapping.

In 1960, A. H. Stone [37] discussed the conditions under which the property of being an H-closed space is hereditary. Recently this property has been extended to non-Hausdorff spaces by C. T. Scarborough and A. H. Stone [30], C. T. Liu [21], and M. K. Singal and Asha Rani [34]. According to Scarborough and Stone, a space is an $H(i)$ space iff every open cover has a finite subfamily, the closures of whose members cover the space. Such spaces have been called "generalised absolutely-closed spaces" by C. T. Liu and "almost-compact spaces" by M. K. Singal and Asha Rani. Many results which are true for H-closed spaces are also true for almost compact spaces. For example, a regular almost compact space is compact; a regularly-closed subset of an almost compact space is almost-compact; a space is almost compact iff for every family $\{G_\alpha\}$ of open sets having the finite intersection property, $\bigcap \bar{G}_\alpha \neq \emptyset$; a θ-continuous image of an almost-compact space is almost compact; an arbitrary product of almost-compact spaces is almost-compact. The last result is due to Scarborough-Stone [30]. M. K. Singal and Asha Rani have shown that a space is almost-compact iff every proper regularly-closed subset is almost-compact. Mamta Kar [22] has shown that a weakly continuous image of a compact space is almost-compact. (The term weak-continuity is due to N. Levine [19] and one may note that a weakly-continuous function may not be θ-continuous.) C. T. Liu has characterized almost-compact spaces in terms of filters as follows:

Theorem 1. *For a topological space X, the following are equivalent*:

(a) *X is almost-compact.*

(b) *Every open filter-base on X has a cluster point.*

(c) *Every open ultrafilter on X converges.*

For H-closed spaces the above result has become pretty standard. (See, for example, Bourbaki.) Some more characterizations of H-closed spaces in terms of filters have been given by N. V. Veličko [40] also.

From time to time several other weaker forms of compactness have been studied. We shall now describe some of these weaker forms. For this purpose let us consider the following 24 types of coverings under the head **A**, and 3 types of sub-families under the head **B**.

A

1. Open cover.
2. Regular open cover.
3. Star-finite open cover.
4. Locally-finite open cover.
5. Point-finite open cover.
6. Star-finite regular open cover.
7. Locally-finite regular open cover.
8. Point-finite regular open cover.

Let 1_c, $2_c \ldots$, 8_c stand respectively for a countable open cover, countable regular open cover, ..., countable point-finite regular open cover. Also if $\mathfrak{m}$ be an infinite cardinal, then $1_{\mathfrak{m}}, 2_{\mathfrak{m}}, \ldots, 8_{\mathfrak{m}}$ stand for open covers of card. $\leqq \mathfrak{m}$ of the above eight types.

B

1. Finite sub-cover.
2. Finite sub-family, the interiors of closures of whose members cover the space.
3. Finite sub-family, the closures of whose members cover the space (or equivalently, finite sub-family whose union is dense in the given space).

Let us denote by $P_{\alpha\beta}$ the property: Given a covering of type α ($\alpha = 1, \ldots, 8, 1_c, \ldots, 8_c, 1_{\mathfrak{m}}, \ldots, 8_{\mathfrak{m}}$), there exists a sub-family of type β ($\beta = 1, 2, 3$). For example, P_{11} will stand for the property: given an open cover there exists a finite sub-cover i.e. P_{11} is nothing but compactness. In this way we shall obtain 72 properties, some of which may be equivalent, but all of which are weaker than compactness.

Now $P_{12} = P_{21} = P_{22}$. Spaces having these properties are called nearly-compact spaces [31].

Also, $P_{13} = P_{23}$. Spaces having these properties are almost-compact spaces.

P_{1_c}, and $P_{1_{\mathfrak{m}}}$, are countable compactness and $\mathfrak{m}$-compactness respectively.

Again, $P_{1_c3} = P_{2_c3}$. Such spaces are studied under various names, such as lightly compact [4, 15], weakly-compact [16], feebly-compact [27].

Again, $P_{1_{\mathfrak{m}}3} = P_{2_{\mathfrak{m}}3}$. Both these properties characterize almost-$\mathfrak{m}$-compactness [34].

Also we have

$$
\begin{array}{llll}
P_{1_c 2} = P_{2_c 1} = P_{2_c 2}\,, & & P_{1_{\mathfrak{m}} 2} = P_{2_{\mathfrak{m}} 1} = P_{2_{\mathfrak{m}} 2}\,, & \\
P_{32} = P_{61} = P_{62}\,, & & P_{42} = P_{71} = P_{72}\,, & \\
P_{3_c 2} = P_{6_c 1} = P_{6_c 2}\,, & & P_{4_c 2} = P_{7_c 1} = P_{7_c 2}\,, & \\
P_{3_{\mathfrak{m}} 2} = P_{6_{\mathfrak{m}} 1} = P_{6_{\mathfrak{m}} 2}\,, & & P_{4_{\mathfrak{m}} 2} = P_{7_{\mathfrak{m}} 1} = P_{7_{\mathfrak{m}} 2}\,, & \\
P_{41} = P_{4_c 1}\,, & & & \\
P_{73} = P_{43} = P_{4_c 3}\,, & & \text{A. H. Stone [37]}\,, & \\
P_{33} = P_{63}\,, & P_{3_c 3} = P_{6_c 3}\,, & P_{3_{\mathfrak{m}} 3} = P_{6_{\mathfrak{m}} 3}\,, & \\
P_{43} = P_{73}\,, & P_{4_c 3} = P_{7_c 3}\,, & P_{4_{\mathfrak{m}} 3} = P_{7_{\mathfrak{m}} 3}\,. &
\end{array}
$$

Examples exist which show that the properties P_{11}, P_{12}, P_{13}, $P_{1_c 1}$, $P_{1_c 2}$, $P_{1_c 3}$, $P_{1_{\mathfrak{m}} 1}$, $P_{2_{\mathfrak{m}} 1}$, $P_{3_{\mathfrak{m}} 1}$, P_{43} are all distinct. Properties P_{71}, P_{41}, P_{43}, P_{31}, P_{33}, $P_{3_c 1}$, $P_{4_c 1}$, $P_{5_c 1}$, have been studied by Iseki and Kasahara in a series of papers which have appeared in the Proc. of Japan Academy (1957) [13, 14, 15, 16]. We are trying to obtain a complete classification of these properties.

We shall now consider some of these properties in detail.

Nearly-Compact Spaces

We have already stated two characterizations of such spaces. These spaces are also characterized as follows: A space is nearly-compact iff every family of regularly-closed sets having the finite intersection property has non-empty intersection or equivalently every family $\mathscr{F}$ of closed sets, having the property that for each finite sub-family $\{F_i : i = 1, \ldots, n\}$ of $\mathscr{F}$, $\bigcap_{i=1}^{n} F_i^{0-} \neq \emptyset$, has non-empty intersection. These spaces can also be characterized in terms of filters. For this purpose we need the following definitions.

Definition 2. [40] *A point x is called a δ-adherent point of the set P in a space X if the interior of every closed neighbourhood of the point x intersects P.*

Definition 3. [40] *The set $[P]_\delta$ of all δ-adherent points of the set P is called the δ-closure of the set P.*

Definition 4. [40] *A point x is a δ-adherent point of a filter iff*

$$x \in \bigcap\{[F]_\delta : F \in \mathscr{F}\}\,.$$

Theorem 2. *For a topological space $(X, \mathscr{T})$, the following are equivalent:*

(i) *$(X, \mathscr{T})$ is nearly-compact.*

(ii) *Every filter in X has a δ-adherent point.*

(iii) *Every ultrafilter in X δ-converges.*

Evidently, every nearly-compact space is almost-compact and every compact space is nearly-compact. Examples exist which show that a space may be almost-compact without being nearly-compact and that a space may be nearly-compact without being compact. Nearly-compact spaces coincide with compact spaces in the case of semi-regular spaces, and with almost-compact spaces in the case of almost-regular spaces [31] or extremally disconnected spaces.

Definition 5. [35] *A space $(X, \mathscr{T})$ is said to be almost-regular iff for every regularly-closed set A and a point $x \notin A$, there exist disjoint open sets G and H such that $x \in G$ and $A \subset H$.*

Every almost-regular Hausdorff space is Urysohn [35] and every nearly-compact Hausdorff space is almost-regular [31]. Therefore every nearly-compact Hausdorff space is Urysohn. Also every Urysohn almost-compact space is almost-regular [26, 35], and consequently a Urysohn almost-compact space is nearly-compact. Thus, nearly-compact Hausdorff spaces are equivalent to almost-compact Urysohn spaces. Since the property of being minimal Hausdorff implies semi-regularity, therefore a minimal-Hausdorff nearly-compact space is compact. A minimal-Hausdorff space need not be nearly-compact and every minimal nearly-compact space is compact.

For the main results about such spaces we need some definitions.

Definition 6. [33] *A function $f : X \to Y$ is said to be almost-continuous iff for each $x \in X$ and a neighbourhood U_y of $y = f(x)$ there exists a neighbourhood V_x of x such that $f(V_x) \subset \overline{U}_y^0$.*

Definition 7. [33] *A function $f : X \to Y$ is said to be almost open iff the image of every regularly-open set is open.*

Theorem 3. [31] *An almost-continuous almost-open image of a nearly-compact space is nearly-compact.*

Theorem 4. [31] *An almost-continuous image of a compact space is nearly-compact.*

Definition 8. [20] *A function $f : X \to Y$ is said to be strongly continuous iff $f(\overline{A}) \subset f(A)$ for every subset A of X.*

Theorem 5. [31] *A strongly continuous image of an almost compact space is compact.*

In this connection it may be noted that on an infinite nearly-compact space, it is not possible to define a one-to-one mapping which is both strongly-continuous and quasi-compact, for the range of a strongly-continuous quasi-compact map is a discrete space which cannot be nearly-compact.

Theorem 6. [31] *An open set $A \subset X$ of a topological space $(X, \mathscr{T})$ is nearly-compact iff every $\mathscr{T}$-open cover $\mathscr{C}$ admits a finite sub-family $\{C_i : i = 1, \ldots, n\}$ such that $A \subset \bigcup_{i=1}^{n} \bar{C}_i^0$.*

Theorem 7. [31] *Every regularly-closed subset of a nearly-compact space is nearly-compact.*

Theorem 8. *A space $(X, \mathscr{T})$ is nearly-compact iff every $\mathscr{T}$-open cover $\mathscr{C}$ of every regularly-closed set A has a finite sub-family $\{C_i: i = 1, 2, \ldots, n\}$ such that $A \subset$ $\subset \bigcup_{i=1}^{n} \bar{C}_i^0$.*

Theorem 9. [31] *Every product of nearly-compact spaces is nearly-compact.*

Lightly-Compact Spaces

Lightly-compact spaces have been studied by R. W. Bagley, E. H. Connell and J. D. McKnight, Jr. [4]. A space is said to be lightly-compact if every locally-finite family of open sets is finite. A space is said to be weakly-compact [16] iff to every pairwise disjoint infinite family of open sets O_α of X, there corresponds a point $x \in X$ such that each neighbourhood V of x meets infinitely many O_α. S. Kasahara [15, Th.2] has shown that a space is weakly-compact iff every locally-finite family of open sets is finite. Thus, weak-compactness and light-compactness are equivalent notions. For these spaces, the following results are known.

Theorem 10. *For any topological space S, the following are equivalent:*

(a) *S is lightly-compact.*

(b) [13] *For any countable family of non-empty open sets G_n of S, having the finite intersection property, $\bigcap_{n=1}^{\infty} \bar{G}_n \neq \emptyset$.*

(c) [13] *For any countable family of closed sets F_n of S having the finite intersection property, if* Int $F_n \neq \emptyset$ *then* $\bigcap_{n=1}^{\infty} F_n \neq \emptyset$.

(d) [13] *For every decreasing sequence of non-empty open sets* $\{G_n\}$, $\bigcap_{n=1}^{\infty} \bar{G}_n \neq \emptyset$.

(e) [13] *For every decreasing sequence of closed sets* $\{F_n\}$ *such that* $\text{Int } F_n \neq \emptyset$, $\bigcap_{n=1}^{\infty} F_n \neq \emptyset$.

(f) [13] *For any regularly-open set* U *containing the intersection of a decreasing sequence of closed sets* F_n $(n = 1, 2, \ldots)$, *there is a closed set* F_{n_0} *such that the interior of* F_{n_0} *is contained in* U.

(g) [4] *Every countable, locally-finite, disjoint collection of open sets is finite.*

(h) [4] *If* $\mathcal{U}$ *is a countable open covering of* S *and* A *is an infinite subset of* S, *then the closures of some members of* $\mathcal{U}$ *contain infinitely many points of* A.

(i) [4] *If* $\mathcal{U}$ *is a countable open covering of* S, *then there is a finite sub-collection of* $\mathcal{U}$, *whose closures cover* S.

(j) [4] *Every proper regularly-closed set is lightly-compact.*

Theorem 11. [13] *Any continuous image of a lightly-compact space is lightly-compact.*

Hewitt [11] introduced the concept of pseudo-compactness. A space is said to be pseudo-compact iff every real-valued continuous function is bounded. Every lightly-compact space is pseudo-compact. The converse may however be not true in general, but in the case of completely-regular spaces they coincide. In [14], it is shown that every normal T_1 pseudo-compact space is countably-compact. Therefore, in a normal T_1-space light compactness, pseudo-compactness and countable compactness coincide.

A space X is said to have Michael property [25] if every open covering of X has a refinement which is the union of countably many locally-finite collection of open sets. For lightly-compact spaces, Lindelöf property is equivalent to the Michael property. Thus, a lightly-compact space satisfying Michael property is almost-compact. A space is compact iff it is paracompact and lightly-compact. A space is countably-compact iff it is countably-paracompact and lightly-compact.

The following results about lightly-compact spaces are due to R. W. Bagley, E. H. Connell and J. D. McKnight Jr. [4].

Theorem 12. *A* T_1*-space* X *is compact iff it is lightly-compact, has the Michael property and each point of* X *has a neighbourhood whose boundary is countably-compact.*

Theorem 13. *Let* X *and* Y *be lightly-compact and let* X *satisfy the following property: For each infinite collection* $\mathcal{V}$ *of open sets of* X *which is not locally-finite,* *there is a point $x \in X$ and an infinite sub-collection $\mathcal{U} \subset \mathcal{V}$ such that each neighbourhood of x intersects all but finitely many elements of $\mathcal{U}$. Then $X \times Y$ is lightly-compact.*

Theorem 14. *If X is first-countable, then $X \times Y$ is lightly-compact iff both X and Y are lightly-compact.*

Since in a completely-regular space, lightly-compact spaces are the same as pseudo-compact spaces, we have the following: If X and Y are completely-regular and X is first-countable, then $X \times Y$ is pseudo-compact and a regularly-closed subset of a completely-regular pseudocompact space is pseudo-compact. These results have also been obtained by Z. Frolík [9].

Lightly-compact spaces play the same role in the theory of minimal E_1-spaces as the almost-compact spaces in the theory of minimal-Hausdorff spaces. (A space is said to be an E_1-space [3] iff every point is expressible as a countable intersection of closed neighbourhoods.)

Theorem 15. [32] *Every lightly-compact semi-regular space is minimal E_1.*

Z. Frolík [10] and R. M. Stephenson [36] have also done some work on lightly-compact spaces.

Almost-$\mathfrak{m}$-Compact Spaces

A space X is said to be almost-$\mathfrak{m}$-compact iff each open covering of X of cardinality $\leqq \mathfrak{m}$ ($\mathfrak{m}$ being an infinite cardinal) has a finite sub-family whose closures cover X.

A space is almost-compact if it is almost-$\mathfrak{m}$-compact for a cardinal not less than the cardinality of an open base. In view of this and of the fact that a regular space is compact if it is almost-compact, we have the following: A regular space having a base of cardinality $\leqq \mathfrak{m}$ is almost-$\mathfrak{m}$-compact if it is compact.

The following properties are equivalent to almost-$\mathfrak{m}$-compactness:

(1) Every open covering of X of cardinality $\leqq \mathfrak{m}$ has a finite sub-family whose union is dense in X.

(2) Each neighbourhood covering of X of cardinality $\leqq \mathfrak{m}$ has a finite sub-family the closures of whose members cover X.

(3) If $\mathcal{F}$ be a family of closed subsets of X of cardinality $\leqq \mathfrak{m}$ with the property that $\bigcap\{F : F \in \mathcal{F}\} = \emptyset$ and $F^0 \neq \emptyset$ for every $F \in \mathcal{F}$, then there exists a finite sub-family $\{F_i : i = 1, \ldots, n\}$ such that $\bigcap_{i=1}^{n} F_i^0 = \emptyset$.

(4) Every family $\{G_\alpha\}_{\alpha \in I}$ of open subsets of X of cardinality $\leqq \mathfrak{m}$ and having

the property that $\bigcap_{\alpha \in I} \bar{G}_\alpha = \emptyset$ has a finite sub-family $\{G_i : i = 1, 2, \ldots, n\}$ such that $\bigcap_{i=1}^{n} G_i = \emptyset$.

(5) Every open-$\mathfrak{m}$-filter base in X has a non-empty adherence. (An open-$\mathfrak{m}$-filter is an open filter which has a base of cardinality $\mathfrak{m}$).

(6) Every open-$\mathfrak{m}$-filter base in X has an open refinement which converges to a point of x.

(7) Every open-$\mathfrak{m}$-net in X has an adherent point. (An open-$\mathfrak{m}$-net is defined to be a function on a directed set of cardinality $\leqq \mathfrak{m}$ whose range is a family of open subsets of X, and a set A is said to be an adherent point of an open-$\mathfrak{m}$-net $\{A_\alpha\}_{\alpha \in D}$ if the net is frequently in each neighbourhood M of each point of A, i.e. given $\alpha \in D$ there exists an $\alpha_i \in D$ such that $\alpha_i \geqq \alpha$ and $A_{\alpha_i} \cap M \neq \emptyset$.

It is known that a space X is almost-$\mathfrak{m}$-compact iff every proper regularly-closed subset of X is almost-$\mathfrak{m}$-compact. A mapping $f : X \to Y$ is said to be irreducible if no proper closed subset of X is mapped onto the whole space Y. If $f : X \to Y$ is a continuous closed irreducible mapping of a space X onto an almost-$\mathfrak{m}$-compact space Y such that $f^{-1}(y)$ is $\mathfrak{m}$-compact for each point $y \in Y$, then X is almost-$\mathfrak{m}$-compact. Also, every weakly-continuous image of an $\mathfrak{m}$-compact space is almost-$\mathfrak{m}$-compact and the product of an almost-$\mathfrak{m}$-compact space with a compact space is almost-$\mathfrak{m}$-compact.

As mentioned earlier, some of the other properties have been studied by K. Iséki and S. Kasahara in a series of papers which appeared in the Proc. of Japan Academy 1957 and also by Z. Frolík [10]. The main results about such spaces are the following:

Theorem 16. *For a regular space S following properties are equivalent:*

(1) *S is lightly-compact.*

(2) *Every point-finite countable infinite open covering has a proper sub-family such that the union of closures of the elements of the family is S.*

(3) *Every point-finite countable infinite open covering has a proper sub-family such that the closures of the unions of elements of the family is S.*

(4) *Any infinite locally-finite family $\{O_\alpha\}$ of open sets of S has a finite sub-family whose union contains every O_α.*

(5) *Every locally finite open covering of S has a finite sub-covering.* $[P_{41}]$

(6) *Every locally finite open covering of S has a finite sub-family whose union is dense in S.* $[P_{43}]$

(7) *Any locally-finite countable open covering of S has a finite sub-family whose union is dense in S.* $[P_{4_c3}]$

(8) *Any locally-finite countable open covering of S contains a finite sub-covering.* $[P_{4_c1}]$

(9) *Every star-finite open covering of S has a finite sub-family whose union is dense in S.* $[P_{33}]$

(10) *Every star-finite open covering of S has a finite sub-covering.* $[P_{31}]$

(11) *Every point-finite open covering has a finite sub-covering whose union is dense in S.* $[P_{53}]$

(12) *Every point-finite countable open covering has a finite sub-covering whose union is dense in S.* $[P_{5_c3}]$

Theorem 17. *The following properties of a completely-regular space S are equivalent:*

(1) *S is lightly-compact.*

(2) *S is pseudo-compact.*

(3) *Every locally-finite open (countable) covering has a finite sub-covering.* $[P_{41}, P_{4_c1}]$

(4) *Every star-finite open (countable) covering has a finite sub-covering.* $[P_{31}, P_{3_c1}]$

(5) *Every locally finite open (countable) covering of S has a finite sub-family whose union is dense in S.* $[P_{43}, P_{4_c3}]$

(6) *Every star-finite open (countable) covering of S has a finite sub-family whose union is dense in S.* $[P_{33}, P_{3_c3}]$

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