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TWO RESULTS CONCERNING BICONNECTED SETS WITH DISPERSION POINTS

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By a *connected* space I understand a connected space (i. e. space which is not a sum of two nonvacuous, disjoint and closed subsets) containing at least two distinct points. A space X is said to be *biconnected* if it is connected and is not a sum of two nonvacuous, disjoint and connected subsets.

The concept of a biconnected set was introduced by B. KNASTER and C. KURATOWSKI [2]. All their examples contain the so called *dispersion point* (i. e. a point p contained in every connected subset). No connected space can have more than one dispersion point [1]. Using the continuum hypothesis, however, E. W. MILLER [6] proved, that there exists a biconnected set which contains no dispersion point.

In what follows I only consider biconnected spaces with dispersion points. Many interesting results are known about them [1], [2], [3], [4], [5], [7] and [8]. I want to supply them with two new results.

I. *For every metric, separable and biconnected space Y there exist a biconnected space X with a dispersion point and a continuous function f that maps X onto Y.*

A space X is said to be *minimally biconnected* if it is a biconnected space with a dispersion point p, and every quasi-component of $X - (p)$ consists of exactly one point.

B. KNASTER [4] constructed minimally biconnected spaces of arbitrary dimension $n = 1, 2, \ldots$ J. H. ROBERTS [7] proved that the set R of all rational points of Hilbert space is homeomorphic with the plane minimally biconnected set whose dispersion point is removed.

Knaster posed the following problem: does there exist a biconnected set with a dispersion point which contains no minimally biconnected set?

If the continuum hypothesis is true, the answer is affirmative. Namely, I prove that:

II. If the continuum hypothesis is true, there exists a plane biconnected set X with a dispersion point p such that, for every biconnected subset $B \subset X$, the set $B - (p)$ contains $2^{\aleph_0}$ quasi-components each of which is of power $2^{\aleph_0}$.

Remark. The proofs of the above results are contained in an article to appear in "Rozprawy Matematyczne".

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