FOURTH WINTER SCHOOL (1976)

COMPACTNESS AND OTHER QUESTIONS IN SPACES OF UNIFORM MEASURES

by

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I have tried to discuss one particular aspect of so far unexplored duality between locally convex spaces and uniform spaces; viz. questions connected with "free" functors from Unif to LocConv and related questions of weak integrals of vector-valued functions. Uniform and free uniform measures arising in this way generalize several long-studied topics in measure theory (such as σ-additive and separable measures on top. spaces, cylindrical measures, σ-additive measures or σ-algebras). In 1975's Winter School I posed the following two problems:

1) In $\mathfrak{M}_U(X)$ for an arbitrary uniform space X, is it true that any weakly compact set is compact ?

2) Is there a "nice" class $\mathcal{C}$ of uniform spaces such that $\mathfrak{M}_F(X)$ = the Radon measures with a compact support in $\hat{X}$, for any $X \in \mathcal{C}$?

Answers: ad 1) Yes. Much more is true:

a) the same holds for sequential compactness

b) the space $\mathfrak{M}_U$ is always weakly sequentially complete

c) the same holds for vector-valued measures

d) the same holds for (vector-valued) free uniform measures

ad 2) Yes. The class of sub-inversion-closed uniform spaces is coreflective and the equality above holds for any sub-inversion-closed space.

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