Jan Pelant

Infinite uniform dimension

In: Zdeněk Frolík (ed.): Abstracta. 5th Winter School on Abstract Analysis. Czechoslovak Academy of Sciences, Praha, 1977. pp. 79--81.

Persistent URL: http://dml.cz/dmlcz/701094

Fifth winter school

Infinite uniform dimension

Jan Pelant

The investigation of a generalization of the notion of large uniform dimension Δd, which is introduced in [I], was presented. The large uniform dimension $\Delta d(X,\mathcal{U})$ of a uniform space $(X,\mathcal{U})$ is said to be $\leq n$ provided every $\mathcal{U}$-uniform cover of X has a uniform refinement of dimension at most n (i.e. at most $n+1$ members of this refinement have a non-empty intersection), (see [I]). The following theorem is proved in [I]:

Theorem: Let $(X,\mathcal{U})$ be a uniform space. The following conditions are equivalent:

(1) $\sigma \in \mathcal{U}$ has an n-dimensional uniform refinement

(2) $\sigma \in \mathcal{U}$ has a uniform refinement which is a union of n+1 disjoint subcollections.

Thus the notions of dimension defined by the order of covers (see (1) of Theorem) and that defined by the decomposition into disjoint collections (see (2) of Theorem) coincide in the finite case. Both these notions are generalized: (1) of Theorem gives the notion of a point-finite cover (i.e. a uniform cover is said to be point-finite proveded that each infinite subfamily of that cover does have an empty intersection); (2) of Theorem gives the notion of a σ-disjoint cover (=unions of at most ω disjoint subcollections). Clearly, both these notions give the possibility to extend a classification of uniform spaces (let us remark that this fact represents one of interesting features of uniform spaces as the Stone paracompactness theorem simplifies substantially the case of topological metrizable spaces but this theorem is not valid for uniform metrizable spaces - see [P_2], [Š]).

The question arises whether Theorem can be extended to point-finite covers and σ-disjoint ones. It is proved in [RR], [P_1] that each σ-disjoint uniform cover does have a point-finite uniform refinement. Nevertheless, it is shown in [P_3] that a point-finite uniform cover need not have a σ-disjoint refinement. Moreover, for any infinite cardinal α, there is a uniform metric space which has a base of uniform covers consisting of point-finite covers and does not have any base consisting of α-disjoint covers (α-disjoint = union of less than α disjoint subcollections). The spaces $c_0(M)$ (= bounded real functions on M with countable supports, the metric is given by sup-norm) are these examples and the Erdős-Rado generalization of the Ramsey theorem is used. It has been shown recently that the Erdős-Rado theorem gives the best estimate of the "decomposition" dimension of $c_0(M)$ - it will be published probably in a joint paper with V.Rödl.

References:

[I] Isbell J.R.: Uniform spaces, Mathematical Surveys (12), A.M.S. 1964.

[P_1] Pelant J.: Remark on locally fine spaces, Coment. math. Univ. Car. (1975).

[P_2] Pelant J.: Cardinal reflections and point-character of uniformities, Seminar Uniform Spaces 1973-1974, Praha 1975.

[P_3] Pelant J.: General hedgehogs in general topology, Seminar Uniform Spaces 1975-1976, Praha 1976.

[RR] Reynolds G.D., Rice M.D.: Completeness and covering properties of uniform spaces, to appear in Proc. Lond. Math. Soc.

[Š] Ščepin E.V.: Ob odnoj probleme Isbella (Russian), Dokl. AN SSSR, 1975.