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Cartesian-closed Coreflective Subcategories of Uniform Spaces

by M. D. Rice and G. J. Tashjian

(abstract)

Let Unif be the category of Hausdorff uniform spaces. We study subfamilies $\mathcal{A}$ of Unif whose coreflective hull $co(\mathcal{A})$ is cartesian-closed. For coreflective subcategories of Unif this means that there must exist function spaces Y^X over the hom-sets $U(X,Y)$ making the exponential law $Y^{X\times Z} = (Y^X)^Z$ valid for all spaces X,Y,Z in the category.

We are particularly interested in the function spaces $\widehat{U}(X,Y)$, equipped with the uniformity of uniform convergence, as possible exponential spaces.

For an infinite cardinal α, let $B(\alpha)$ be the class of all uniform spaces which have covering character at most α and which admit all cardinals less than α.

Theorem. Let $\mathcal{A} \subseteq$ Unif and let c be the coreflector to $co(\mathcal{A})$. The following are equivalent:

(1) $co(\mathcal{A})$ is cartesian-closed with exponentials $Y^A = c\widehat{U}(A,Y)$ for all $A \in \mathcal{A}$ and $Y \in co(\mathcal{A})$.

(2) There exists a finitely productive subfamily $\mathcal{A}'$ of locally fine spaces such that $\mathcal{A} \subseteq \mathcal{A}' \subseteq co(\mathcal{A})$.

(3) There exists a finitely productive subfamily $\mathcal{A}'$ of $B(\alpha)$, for some $\alpha \geq \aleph_0$, such that $\mathcal{A} \subseteq \mathcal{A}' \subseteq co(\mathcal{A})$.

If these conditions are satisfied, then the closed unit interval I belongs to $co(\mathcal{A})$ if and only if all spaces in $\mathcal{A}$ are precompact, and $I \notin co(\mathcal{A})$ if and only if all spaces in $\mathcal{A}$ admit $\aleph_0$.

<u>Examples</u>. co(B(α)) is cartesian-closed for each $\alpha \geq \aleph_0$.

Cartesian-closed coreflective subcategories of <u>Unif</u> need not be formed in this way, however. For example, if $\mathcal{A}$ is the class of all precompact proximally discrete uniform spaces, then co($\mathcal{A}$) is cartesian-closed since its product preserves sums and quotients. However, there is no finitely productive family $\mathcal{A}'$ in Unif such that co($\mathcal{A}$) = co($\mathcal{A}'$). Therefore, some of the exponential spaces in this category must differ from $c\hat{U}(X,Y)$.