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Problems

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Problems

A.Hajnal

a/ R. Faudree stated the following problem. Assume G is a countable such that $G\rightarrow(K_{\omega,\omega})^2_2$. Does then $K_\omega\subset G$ hold?

I realized that this follows from an old result of Erdős, Pósa and myself, which says that $G\not\rightarrow(K_{\omega,\omega})^2_2$ holds for every countable graph [1]. Then Faudree asked what can be said about countable graphs G for which $G\rightarrow(P_\infty)^2_2$ holds. Here P_∞ denotes a /one-way/ infinite path. During this conference Rödl gave examples of countable graphs G_r $r\geq 2$ such that $G_r\rightarrow(P_\infty)^2_r$ and $K_{r+1,r+1}\not\subset G_r$. On the other hand, I can prove that if $G\rightarrow(P_\infty)^2_{2r}$ holds for a countable G, then $K_{r,\omega}\subset G$ for $r\geq 2$. The problem remains open if there is a countable G such that $C_4\not\subset G$ and $G\rightarrow(C_4)^2_2$. Note that by an example of Rödl there is such a graph of cardinality $\aleph_1$.

[1] P. Erdős, A. Hajnal, L. Pósa: Strong embeddings of graphs into colored graphs. Coll. Math. Soc. János Bolyai 10. Infinite and finite sets. Keszthely /Hungary/, 1973, 585-595.

b/ Modifying a question of F. Galvin I asked the following problem. For what cardinals $\varkappa$ does there exists a graph $G=\langle\varkappa, E\rangle$ such that $G\rightarrow(\aleph_1)^2_2$ or $G\rightarrow(\aleph_0)^2_\omega$ but this fails to be true for any subgraph G' of cardinality less than $\varkappa$ of G? I can prove, that if $\varkappa>\omega$ is a regular cardinal such that $\forall\lambda<\varkappa$ $\lambda^{\aleph_0}<\varkappa$ and $E_\varkappa$ holds, then there is a graph $G = \langle \varkappa, E \rangle$ such that $G \to (\aleph_1)^2_{\aleph_0}$ but $G' \not\to (\aleph_1)^2_2$ and $G' \to (\aleph_0)^2$ hold for all $G' \subset G$ with $|G'| < \varkappa$. Can such an example exist, say assuming G.C.H. for $\varkappa = \aleph_{\omega+1}$?

PROBLEMS

Problem Which partition properties does the class of finite groups have?

In particular prove the following:

For every finite group G there exists a finite group H satisfying $H \rightarrow (G)^{\mathbb{Z}_2}_2$, i.e. for every 2-coloring of the $\mathbb{Z}_2$-subgroups of H there exists a G-subgroup with all its $\mathbb{Z}_2$-subgroups colored the same.

Problem Prove or disprove:

For every 2-coloring of the positive integers there exist positive integers a,d such that the elements of the arithmetic progression $a, a+d, \ldots, a+d^2$ all are colored the same.

Bernd Voigt

PROBLEMS

We use the notations of the paper J. Lehel: On hypergraph coverings, appeared in this volume.

<u>Conjecture.</u> For any $1 < r < p \leqq n$ every r-uniform hypergraph of order n has a K_p-partition of size not greater than $T(n,p,r)$.

<u>Remark.</u> The conjecture is true for $r=2$, see in B. Bollobás: On complete graphs of different orders, Math.Proc.Philos.Soc. 79(1976), 19-24.

Jenő Lehel

PROBLEMS

Let $\mathbb{L}$, or $\mathbb{K}$, be the ideal of all subsets of the unite real interval I, of Lebesgue measure 0, or of the first Baire category, respectively. In the field $\mathcal{B}$ of all Borel sets in $I \times I$ we define the product $\mathbb{L} \times \mathbb{K}$ by the standard definition: for $A \in \mathcal{B}$, $A \notin \mathbb{L} \times \mathbb{K} \equiv$

$$\equiv \{x_0 \in I \ ; \{x_1 \in I \ ; \ (x_0, x_1) \in A\} \notin \mathbb{K}\} \in \mathbb{L} .$$

The product $\mathbb{K} \times \mathbb{L}$ is defined analogously.

Question: are boolean factor-algebras $\mathcal{B}/\mathbb{L} \times \mathbb{K}$, $\mathcal{B}/\mathbb{K} \times \mathbb{L}$ isomorph ?

Martin Gavalec

PROBLEMS

Definition 1. If $\mathcal{M}_1$ and $\mathcal{M}_2$ are matroids on the same set S then their sum is a matroid $\mathcal{M}_1 \vee \mathcal{M}_2$ on S so that $X \subseteq S$ is independent in $\mathcal{M}_1 \vee \mathcal{M}_2$ if and only if $X = X_1 \cup X_2$ with X_i independent in $\mathcal{M}_i$ for $i=1$ and $i=2$.

Conjecture 1. If $\mathcal{M}_1$ and $\mathcal{M}_2$ are graphic but $\mathcal{M}_1 \vee \mathcal{M}_2$ is not then $\mathcal{M}_1 \vee \mathcal{M}_2$ is not binary.

Definition 2. Weak map is the name of the following partial order among matroids on the same set: $\mathcal{M}_1 \subseteq \mathcal{M}_2$ if and only if every independent subset of $\mathcal{M}_1$ is also independent in $\mathcal{M}_2$.

Conjecture 2. Suppose the equation $\mathcal{A} \vee \mathcal{X} = \mathcal{B}$ is solvable for a given pair $\mathcal{A}$, $\mathcal{B}$, and arrange every solution by the weak map. Then there exists a unique maximal among them.

Remarks. The algorithm of [1] does not help to prove the first conjecture. If $\mathcal{B}$ is graphic, the second conjective is true [2].

References

[1] A. Recski: An algorithm to determine whether the sum of some graphic matroids is graphic, Coll-Math.Soc.J. Bolyai (Algebraic methods in graph theory, Szeged, 1978). North-Holland Publ. Co., Amsterdam 1981.

[2] A. Recski: On the sum of matroids III, Discrete Mathematics, in press.

A. Recski

PROBLEMS

1. If G is a graph such that $G \to (K_{\omega,\omega})^2_2$, does $G \to (K_{\omega,\omega})^2_n$ for all $n < \omega$? for $n=3$?

2. If some metric space of cardinality $\varkappa$ is strongly of measure zero, does it follow that some subset of the real line of cardinality $\varkappa$ is strongly of measure zero? Timothy J. Carlson has shown that this is true if $\varkappa < 2^{\aleph_0}$ or $\varkappa = \aleph_1$. What happens if $\varkappa = 2^{\aleph_0} > \aleph_1$?

3. For a cardinal $\varkappa$, let $P(\varkappa)$ be the statement: every subset of $\varkappa\times\varkappa\times\varkappa$ belongs to the σ-algebra generated by the sets of the form

$$\{(x_1,x_2,x_3) : (x_i,x_j)\in S\}.$$

where $1\le i<j\le 3$ and $S \subseteq \varkappa\times\varkappa$. It is known that

$$\varkappa \le \min\{\aleph_2, 2^{\aleph_0}\} \Rightarrow P(\varkappa) \Rightarrow \varkappa \le 2^{2^{\aleph_0}}.$$

Assume GCH; is $P(\aleph_2)$ true or false?

4. If $G=(V,E)$ is a graph, $\varphi(G)$ is the least cardinal n such that, for any family $(C_x : x\in V)$ of n-element sets, there are elements $a_x\in C_x$ $(x\in V)$ such that $a_x \ne a_y$ whenever $\{x,y\}\in E$. Note that $chr(G) \le \varphi(G) \le col(G)$, where $chr(G)$ is the ordinary chromatic number and $col(G)$ is the Erdős-Hajnal coloring number [P. Erdős and A. Hajnal, On chromatic number of graphs and set-systems,

Acta Math. Acad.Sci.Hungar. 17(1966), 61-99.] For example, if $G = K_{3,3}$, then $chr(G) = 2$, $\varphi(G) = 3$, $Col(G) = 4$. Determine $n = \max\{\varphi(G) : G$ is planar$\}$. Clearly $4 \leq n \leq 6$.

Fred Galvin