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In: Jan Chleboun and Karel Segeth and Tomáš Vejchodský (eds.): Programs and Algorithms of Numerical Mathematics, Proceedings of Seminar. Prague, May 28-31, 2006. Institute of Mathematics AS CR, Prague, 2006. pp. 196–201.

Persistent URL: http://dml.cz/dmlcz/702837

THE APPLICATION OF THE THERMAL BALANCE METHOD FOR COMPUTATION OF WARMING IN ELECTRIC MACHINES*

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Abstract

The paper describes the procedure of the thermal balance method implementation for the computation of warming in electrical machines. Our effort will be focused on the temperature distribution in transformer screening under a stationary load. Since the three-dimensional problem is axially symmetric, it will be reduced by means of the cylindrical coordinates to an elliptic partial differential equation of second order with the Newton boundary conditions on a rectangular domain. Results of numerical tests are presented as well.

1. Introduction

Heat energy is being accumulated in an electrical machine during its operation. Thus, the temperature increase in its different parts depends on the accumulated heat energy. The electrical machine operating temperature is an important feature of a proper functioning and lifespan. The highest (and often also the lowest) operating temperature is prescribed for most of machine components.

These requirements could be reached by limiting the ambient temperature, at which the machine works in and by preventing machine parts warming over specified allowable limits. One of the most effective approaches for solving these problems is the description of spreading heat in electrical machines by means of a mathematical model, which is subsequently investigated. At present, mathematical models are often solved by using a variational formulation (see e.g. [3] and [4]). A one-dimensional problem of heat conduction is solved in [5]. This contribution is focused on the computation of warming of a transformer container screening at a stationary load by means of the thermal balance method.

2. Problem definition

Transformer screening is considered in the form of a thin-wall cylinder and the temperature field is supposed to be rotationally symmetric. Therefore, the warming computation problem can be solved in screening cross section on a two-dimensional closed domain Ω ($R_1 \leq r \leq R_2$, $Z_1 \leq z \leq Z_2$, see Fig. 1).

The temperature field is described by the elliptic partial differential equation of second order (see [3, p. 221])

$$\lambda_r \left(\frac{\partial^2 \vartheta(r,z)}{\partial r^2} + \frac{1}{r} \frac{\partial \vartheta(r,z)}{\partial r} \right) + \lambda_z \frac{\partial^2 \vartheta(r,z)}{\partial z^2} = -q(r,z) \tag{1}$$

*Technical University of Liberec Internal Grant IGS 5130/1 supported this contribution.

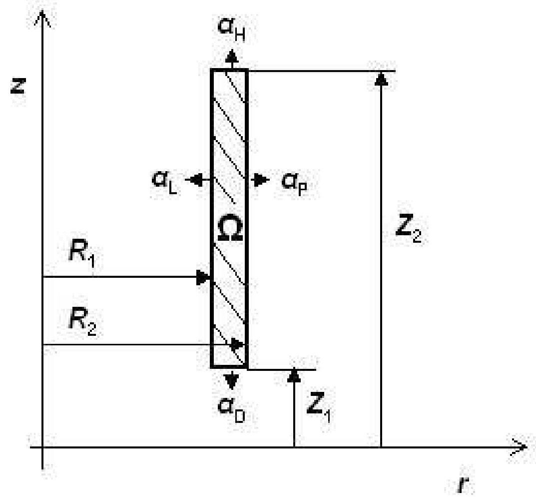

Fig. 1: *Cross section of the transformer screening.*

with the Newton boundary conditions

$$\lambda_r \frac{\partial \vartheta(r,z)}{\partial r} + \alpha_{L,P}\left(\vartheta(r,z) - u(z)\right) = 0 \tag{2}$$

on vertical parts of boundary of Ω and

$$\lambda_z \frac{\partial \vartheta(r,z)}{\partial z} + \alpha_{H,D}\left(\vartheta(r,z) - u(z)\right) = 0 \tag{3}$$

on horizontal parts for appropriate values of r and z. Real values λ_r and λ_z stand for heat conductivities of the material in the r-axis and z-axis directions, respectively; the true solution $\vartheta(r,z)$ denotes screening temperature rise with respect to the surrounding oil temperature. The function $u(z)$ in expressions (2) and (3) allows to respect the variable temperature of oil in the vicinity of screening in the z-axis direction. It is given by the formula $u(z) = Cz$, where C is constant. In expression (1), the function $q(r,z)$ represents the volume density of losses, which is expressed by the following relation:

$$q(r,z) = \delta^2(z)\rho(1 + \alpha_T \vartheta(r,z)), \tag{4}$$

where $\delta(z)$ denotes the density of eddy currents, ρ is the specific resistance of the material used for screening, and α_T is the factor for the dependence of a specific resistance on temperature. In boundary conditions (2) and (3), the constants α_L, α_P, α_H, and α_D stand for the heat transfer coefficients on the left, right, upper, and lower parts of the rectangular domain Ω, respectively.

3. Solving the problem by means of the thermal balance method

Equation (1) can be transformed to a self-adjoint form and after the substitution of the function $q(z)$ from expression (4), the basic equation will be obtained. It describes warming in the cross section Ω of transformer screening:

$$\frac{\partial}{\partial r}\left(\lambda_r r \frac{\partial \vartheta(r,z)}{\partial r}\right) + r\frac{\partial}{\partial z}\left(\lambda_z \frac{\partial \vartheta(r,z)}{\partial z}\right) = -r\delta^2(z)\rho(1+\alpha_T\vartheta(r,z)) \tag{5}$$

with boundary conditions (2) and (3).

In the domain Ω, a regular rectangular mesh will be constructed with increments

$$h_r = \frac{R_2 - R_1}{m} \quad \text{and} \quad h_z = \frac{Z_2 - Z_1}{n}$$

in the r-axis and z-axis directions, respectively, where m and n denotes the number of segments, to which the region is divided in the r-axis and z-axis directions, respectively. Let us denote $r_k = R_1 + kh_r$, $z_s = Z_1 + sh_z$, and $\vartheta_{k,s} = \vartheta(r_k, z_s)$ the warming at the node $[r_k, z_s]$, where $k \in \{0, 1, ..., m\}$, $s \in \{0, 1, ..., n\}$.

Let the point $[r_k, z_s]$ be an internal node in the domain Ω (see Fig. 2). Then equation (5) can be approximated at this node using the following balance of heat:

$$\lambda_r\left(r_k + \frac{h_r}{2}\right)\frac{\vartheta_{k+1,s} - \vartheta_{k,s}}{h_r}h_z + \lambda_r\left(r_k - \frac{h_r}{2}\right)\frac{\vartheta_{k-1,s} - \vartheta_{k,s}}{h_r}h_z +$$
$$+ \lambda_z r_k \frac{\vartheta_{k,s+1} - \vartheta_{k,s}}{h_z}h_r + \lambda_z r_k \frac{\vartheta_{k,s-1} - \vartheta_{k,s}}{h_z}h_r = -r_k\delta^2(z_s)\rho(1+\alpha_T\vartheta_{k,s})h_r h_z. \tag{6}$$

The left-hand side of equation (6) describes the approximate quantity of heat supplied from or delivered to surrounding mesh nodes, the right-hand side expresses approximate waste heat arising in the element that pertains to the node $[r_k, z_s]$.

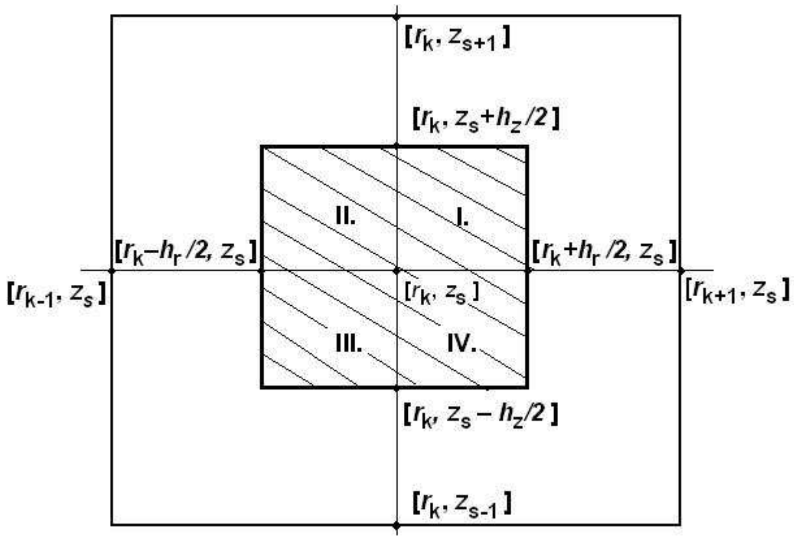

Fig. 2: *The neighborhood of the point* $[r_k, z_s]$.

Fig. 2 shows four parts (I, II, III, and IV) of a square neighborhood of a node $[r_k, z_s]$. Clearly, if the node lies on the boundary of Ω or at the corner then the neighborhood consists of two or one part, only. For the boundary nodes, boundary conditions (2) or (3) will be used to determine the thermal balances. For instance, as long as the neighborhood of the boundary point $[r_k, z_s]$ consists of parts III and IV only, we obtain by means of thermal balances the following equation:

$$\lambda_r\left(r_k+\frac{h_r}{2}\right)\frac{\vartheta_{k+1,s}-\vartheta_{k,s}}{h_r}\frac{h_z}{2}+\lambda_r\left(r_k-\frac{h_r}{2}\right)\frac{\vartheta_{k-1,s}-\vartheta_{k,s}}{h_r}\frac{h_z}{2}+$$
$$+\lambda_z r_k\frac{\vartheta_{k,s-1}-\vartheta_{k,s}}{h_z}h_r-\alpha_H r_k\left(\vartheta_{k,s}-u(z_s)\right)h_r=-r_k\delta^2(z_s)\rho(1+\alpha_T\vartheta_{k,s})h_r\frac{h_z}{2}. \quad (7)$$

Let us set $h = \max(h_r, h_z)$. Then we make the $O(h^2)$-order error by approximating equation (5) in the internal node $[r_k, z_s]$, since central differences are used. In boundary nodes we make the $O(h)$-order error in the approximation (see [2, p. 277]), because the difference

$$\lambda_z r_k\frac{\vartheta_{k,s+1}-\vartheta_{k,s}}{h_z},$$

for example, is substituted in equation (7) by the expression

$$-\alpha_H r_k(\vartheta_{k,s}-u(z_s))$$

from relation (3). This low accuracy is quite sufficient in our case, since all physical constants suffer from large uncertainties. By equations of type (6) and (7) in all mesh points, we obtain a system of linear algebraic equations with a band symmetric and positive definite matrix (for practically used values of physical quantities from equations (1), (2), and (3)). The Choleski decomposition algorithm (see [1]) was used to solve the associated system.

4. One-dimensional problem of heat conduction

For a one-dimensional heat conduction, an analytical solution can be determined and compared with an approximate solution obtained by means of the thermal balance method. Let us examine the case, when a one-dimensional heat conduction is considered in the r-axis direction. The heat transfer coefficient is nonzero only on the vertical part of the boundary of Ω (i.e. $\alpha_L = 0$, $\alpha_P \neq 0$), the current density δ is constant, and $\alpha_T = 0$. Then, equation (1) attains a simple form:

$$\lambda_r\left(\frac{\partial^2\vartheta(r)}{\partial r^2}+\frac{1}{r}\frac{\partial\vartheta(r)}{\partial r}\right)=-q, \quad (8)$$

where $q = \delta^2\rho$.

For the solution ϑ of problem (8) in the interior point r we have:

$$\vartheta_r = X\left\{\frac{2\lambda_r}{\alpha_P R_2}Y+1-\left(\frac{r}{R_2}\right)^2-2\left(\frac{R_1}{R_2}\right)^2\ln\frac{R_2}{r}\right\}. \quad (9)$$

The temperature at boundary nodes is given by:

$$\vartheta_{R_1} = X\left\{\left(\frac{2\lambda_r}{\alpha_P R_2}+1\right)Y - 2\left(\frac{R_1}{R_2}\right)^2 \ln\frac{R_2}{R_1}\right\}, \tag{10}$$

$$\vartheta_{R_2} = \vartheta_{R_1} + q\frac{R_1^2}{4\lambda_r}\left[2\ln\frac{R_2}{R_1}+1-\left(\frac{R_2}{R_1}\right)^2\right], \tag{11}$$

where

$$X = q\frac{R_2^2}{4\lambda_r}, \quad Y = 1-\left(\frac{R_1}{R_2}\right)^2.$$

The proof of relations (9)–(11) is based on the transformation of equation (8) to the form

$$\frac{\partial}{\partial r}\left(r\frac{\partial\vartheta}{\partial r}\right) = -\frac{qr}{\lambda_r},$$

repeatedly using the integration with respect to r and applying the conditions $\alpha_P \neq 0$ and $\alpha_L = 0$.

Table 1 lists approximate values of temperature rise computed numerically by means of the thermal balance method and the values obtained through analytical formulae (9)–(11) for the following input values: $q = 10^5\,\mathrm{W/m^3}$, $R_1 = 1\,\mathrm{m}$, $R_2 = 1.1\,\mathrm{m}$, $\alpha_P = 50\,\mathrm{W/m^2K}$, $\alpha_L = \alpha_H = \alpha_D = 0$, and $\lambda_r = 1\,\mathrm{W/mK}$.

$h_r[m]$	ϑ_{R_1}[K] $R_1 = 1$[m]		ϑ_r[K] $r = 1.05$[m]		ϑ_R[K] $R_2 = 1.1$[m]	
	approx.	exact	approx.	exact	approx.	exact
0.05	673.33		551.37		190.91	
0.025	674.88	675.37	552.15	552.41	190.91	190.88
0.0167	675.17		552.29		190.91	

Tab. 1: *One-dimensional heat transfer, the comparison of the exact and approximate values of warming.*

5. Numerical example

By means of the above mentioned thermal balance method, the real-live problem was solved that involved finding the warming in aluminium transformer screening with the following input parameters: $R_1 = 0.86\,\mathrm{m}$, $R_2 = 0.868\,\mathrm{m}$, $Z_1 = 0.8864\,\mathrm{m}$, $Z_2 = 2.51\,\mathrm{m}$, $\lambda_r = \lambda_z = 220\,\mathrm{W/mK}$, $\rho = 0.3\times10^{-7}\Omega\,\mathrm{m}$, $\alpha_L = \alpha_P = \alpha_H = \alpha_D = 50\,\mathrm{W/m^2K}$, $\alpha_T = 0.00409\,\mathrm{K^{-1}}$, $C = 10\,\mathrm{K/m}$ (C is the costant appearing in the definition of the function $u(z)$ in expressions (2) and (3) in Section 2). The domain Ω is divided into 2 segments ($h_r = 0.4\times10^{-2}\,\mathrm{m}$) in the r-axis direction and subsequently to 16, 32 and 64 segments ($h_z = 0.10148\,\mathrm{m}$, $h_z = 0.050738\,\mathrm{m}$, $h_z = 0.025369\,\mathrm{m}$) in the z-axis direction. The current density $\delta(z)$ is given by means of 19 values between $0.2498\times10^5\,\mathrm{Am^{-2}}$ and $0.3508\times10^7\,\mathrm{Am^{-2}}$, the current density

		$R_1 = 0.86$ [m]	$r = 0.864$ [m]	$R_2 = 0.868$ [m]
$Z_2 = 2.51$ [m]	$h_z = 0.101480$ [m]	35.790	35.795	35.790
	$h_z = 0.050738$ [m]	30.908	30.911	30.908
	$h_z = 0.025369$ [m]	29.444	29.446	29.444
$z = 1.6982$ [m]	$h_z = 0.101480$ [m]	19.481	19.482	19.481
	$h_z = 0.050738$ [m]	19.467	19.468	19.467
	$h_z = 0.025369$ [m]	19.466	19.467	19.466
$Z_1 = 0.8864$ [m]	$h_z = 0.101480$ [m]	12.607	12.609	12.607
	$h_z = 0.050738$ [m]	12.764	12.765	12.764
	$h_z = 0.025369$ [m]	12.809	12.811	12.809

Tab. 2: *The screening temperature rise (in K) for selected nodes at $h_r = 0.004$ [m].*

at the other node points is computed by means of linear interpolation. Table 2 lists approximate values of temperature rise $\vartheta_{k,s}$ (at chosen nodes) computed numerically using the thermal balance method.

6. Conclusion

The problem (1)–(3) for specific values of transformer screening was solved by means of the above mentioned thermal balance method. The described method of solving is relatively simple, but still allows to obtain an approximate solution, which is sufficiently exact in technical practice. In numerical calculations of warming in transformer screening, the domain Ω was divided only to 2 segments in the r-axis direction (in view of the thin-wall cylindrical area of screening). The value of the increment $h_z = 0.05\,\mathrm{m}$ in the z-axis direction was sufficient. The described procedure can be used for the examination of transformer parts at various load levels during the development of transformer designs.

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