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UNCERTAINTIES IN MEASUREMENT OF THERMAL TECHNICAL CHARACTERISTICS OF BUILDING INSULATIONS*

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1. Thermal technical characteristics of building insulations

Most thermal insulation materials used in civil engineering, namely those applied as parts of layered constructions in buildings, have a complicated porous micro- and macrostructure; thus the reliable prediction of their thermal insulation and accumulation properties is rather difficult. Technical standards require the so-called thermal stability, which means in practice i) the preservation of nearly constant temperature $T(t)$ in time $t \geq 0$ in the interior of the building, independent of quasi-periodic (day and year) climatic cycles, and ii) very slow changes of time derivatives of $T(t)$; moreover iii) the minimization of energy consumption by heating (in winter) and air conditioning (in summer) should be guaranteed. The general description of physical processes in porous materials comes from the classical conservation laws for mass, momentum, and energy and contains: i) the heat conduction, convection, and radiation (Fourier equation); ii) the partially irreversible propagation of moisture in various phases (as air humidity, liquid water and ice) and, possibly, of other contaminants; and iii) the compressible viscous air flow in rooms and through walls, roofs, etc. (Navier-Stokes equations). In the corresponding initial and boundary problems for systems of partial differential equations of evolution we need to know a lot of thermal technical characteristics, especially of a) the heat conduction; b) the heat convection; c) the heat radiation; d) the pore space and its availability for air, moisture, and contaminants; and e) the air flow in rooms, walls, roofs, etc. Typically such characteristics depend on T and other quantities, e. g. on the moisture content (and its phase) in applied materials.

One of the research directions at the Faculty of Civil Engineering of the Brno University of Technology is the development of ecological insulation materials, based on the wood waste. The crucial step of such experimental research is some reliable estimate of the basic thermal technical characteristics, at least those denoted as a). Under the assumption that the material is homogeneous and isotropic (due to the technology of its composition) the heat conduction can be described using three constants only: i) the material density ρ, ii) the heat conduction factor λ, iii) the thermal capacity (specific heat) c. Consequently the thermal insulation ability is determined by λ, and the thermal accumulation ability is determined by c, see [6, pp. 52, 57]. Frequently we shall apply the notation $\zeta = c\rho$.

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2. Laboratory measurements

The classical approach to the simulation of heat transfer is to solve the classical differential equation (see [2, p. 263])

$$\zeta\, \partial T/\partial t - \lambda\, \partial^2 T/\partial x^2 = r \tag{1}$$

(with some "effective characteristics" λ, ζ) in two variables: in t and in one space coordinate x; r represents (in general, as a function of t) the generated heat (per length). The setting of ρ is easy: it can be obtained as the ratio mass / volume. The identification of λ comes usually from standard experiments with the stationary heat transfer; in this case the first additive (time-dependent) term in (1) is missing. However, c must be obtained in another way, using various calorimeters where the contact with water is needed; this brings a danger of mismatched results caused by humidity in pores.

In [5], a new approach to measurements has been suggested: both λ and c (or ζ because ρ in known) can be obtained from an experiment that, unlike the standard experiment for determining λ, takes a non-stationary heat transfer into consideration. A more advanced numerical analysis is then needed for the simultaneous identification of both characteristics.

Fig. 1 presents the scheme of the original measurement device; we use the following notation: 1 – the thermal insulation (foam polystyrene blocks), 2 – two aluminium plates, just the lower one is heated, 3 – the tested sample (with unknown λ and ζ), 4 – the highlighting of the direction of thermal flow, 5 – two temperature sensors, 6 – the data recorder. Fig. 2 shows such a device in practice.

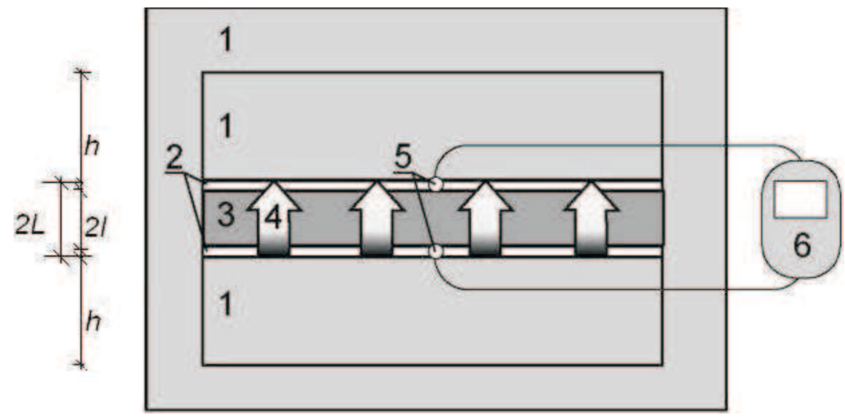

Fig. 1: *Principle of simultaneous measurement of* λ *and* c*.*

For the reliable identification of λ and c the technical standards require the analysis of uncertainties of measurements, see [3]. We shall see that in our approach such analysis will be available and using the same numerical technique as is used for the identification of the deterministic values of λ and c.

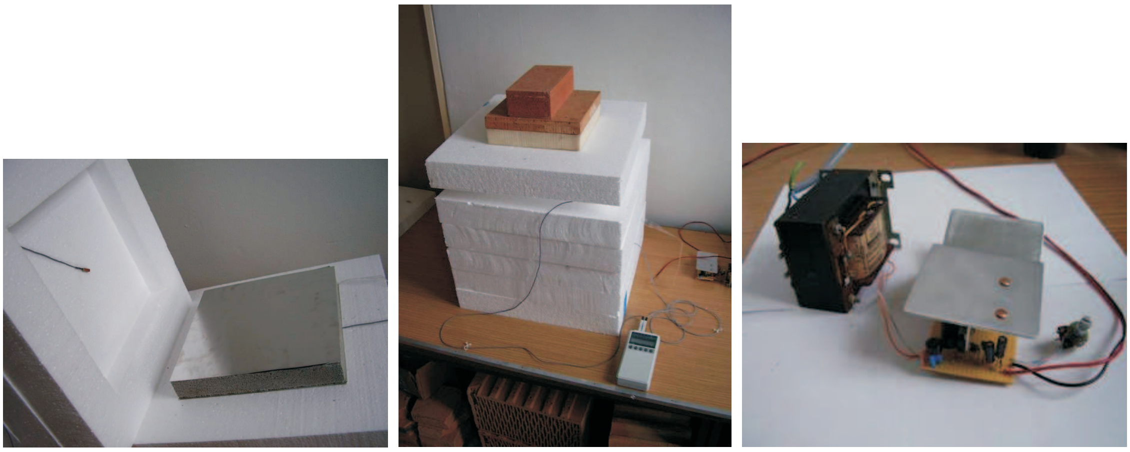

Fig. 2: *Measurement of basic thermal technical characteristics in the laboratory.*

3. An inverse problem in heat conduction

Let l, L and H be three positive numbers, $0 < l < L \ll H$, $L - l \ll l$. The technical interpretation of l, L and H is evident from the simplified scheme at Fig. 1: $2l$ is the thickness of the measured material sample, $L - l$ is the thickness of each of two aluminium plates, $2H = 2(L + h)$ denotes the total size of the whole insulated system where h is the thickness of each of two massive insulation blocks. Let us introduce the set $A = \{(-H, -L), (-L, -l), (-l, l), (l, L), (L, H)\}$ of couples of x-coordinates, too.

The above sketched classical formulation of the heat equation (1) can be (especially for layered structures) converted into the weak formulation

$$\zeta(\varphi, \dot{T}) + \lambda(\varphi', T') = r(\varphi, 1) + \varphi(a_+)q_+ - \varphi(a_-)q_- \,, \tag{2}$$

satisfied for every test function φ from the Sobolev space $W^{1,2}(a_-, a_+)$ where (a_-, a_+) are elements of A, with (a priori unknown) interface heat fluxes $q_-(t)$ and $q_+(t)$; q_- and q_+ here is the brief notation for heat fluxes at the interfaces $x = a_-$ and $x = a_+$ The dot symbol in (2) is reserved for partial derivatives by t, the prime symbol for partial derivatives by x, $(\psi, \widetilde{\psi})$ are scalar products in the Lebesgue space $L^2(a_-, a_+)$ for any ψ and $\widetilde{\psi}$ from this space, i. e.

$$(\psi, \widetilde{\psi}) = \int_{a_-}^{a_+} \psi(x)\, \widetilde{\psi}(x)\, \mathrm{d}x \,.$$

We suppose that for $t = 0$ the temperature $T_0(x)$ is known everywhere (for $-H \leq x \leq H$), thus the initial condition $T(x, 0) = T_0(x)$ can be prescribed. We also assume that the whole system is perfectly insulated from the external environment (the experiment cannot be too long in practice), thus for $x = -H$ and $x = H$ no heat fluxes q_- or q_+ are considered. The unknowns are $T(x, t)$ everywhere for any positive time t, and (time-independent) λ and ζ only for $-l \leq x \leq l$ (inside the tested sample).

Alternatively the weak formulation (2) could be rewritten for $(a_-, a_+) = (-H, H)$ with test functions $\varphi \in W^{1,2}(-H, H)$; then all q_- and q_+ seemingly vanish. However, in such notation all scalar products $\lambda(.\,,.)$ and $\zeta(.\,,.)$ would obtain more complicated forms $(.\,,\lambda\,.)$ and $(.\,,\zeta\,.)$ with piecewise continuous functions λ and ζ, whose values are not known a priori everywhere. Consequently it is not possible to avoid all explicit calculations of q_- and q_+, at least those corresponding to $x = -l$ and $x = l$.

The identification of λ and ζ is based on the comparison of the temperatures $T(-l, t_s)$ and $T(l, t_s)$ from numerical simulation with the temperatures

$$T_{s-} \approx T(-l, t_s)\,, \qquad T_{s+} \approx T(l, t_s)\,, \tag{3}$$

obtained from sensors in a finite integer number S of times t_s, $s \in \{1, \ldots, S\}$; in such discrete time steps the heat generator is able to guarantee the constant values $r_s = r(t_s)$ of r from the right-hand side of (2) inside the whole heated plate (where $l < x < L$). Unlike the much more general approach of [4], thanks to the very simple arrangement of the experiment, we are able to apply the semi-analytical Fourier method here. Following [1, pp. 229, 256], instead of $T(x, t)$ in (2) we can consider $T_N(x, t)$ with a large integer N (theoretically $N \to \infty$) in the form

$$T_N(x, t) = T_N(x, t_*) + \sum_{n=0}^{N} \varphi_n(\tilde{x})\, \alpha_n(t - t_*) \tag{4}$$

with $\tilde{x} = (x - a_-)/(a_+ - a_-)$ and $0 < t_* < t$, and attempt (once the system $\varphi_n(\tilde{x})$, $0 \le \tilde{x} \le 1$, $n \in \{1, \ldots, N\}$, is available) to find the approximate solution $T_N(x, t)$ of (2); in practice we are allowed to set (step by step) $t_* = t_{s-1}$ and $t = t_s$.

The one-dimensional discretization in the variable x enables us to apply the method of lines: inserting $T_N(x, t)$ from (4) into (2), we obtain the system of $N + 1$ ordinary differential equations, whose general form is

$$a\zeta M\dot{\alpha} + a^{-1}\lambda K\alpha = \beta_+ q_+ - \beta_- q_- + arg\,, \tag{5}$$

where (for simplicity) $a = a_+ - a_-$ and

$$\alpha(\tau) = [\alpha_0(\tau), \ldots, \alpha_N(\tau)]^{\mathrm{T}}, \quad \beta_- = [\varphi_0(0), \ldots, \varphi_n(0)]^{\mathrm{T}}, \quad \beta_+ = [\varphi_0(1), \ldots, \varphi_n(1)]^{\mathrm{T}}.$$

Let us remind that (5) must be formulated for each couple $(a_-, a_+) \in A$ separately and that $r \ne 0$ for $(a_-, a_+) = (l, L)$ only. The concrete form of the square "mass" and "stiffness" matrices M, K, generated by (φ_m, φ_n) and (φ'_m, φ'_n) with $m, n \in \{0, \ldots, N\}$, and of the "load" vector g, generated by $(\varphi_m, 1)$ with $m \in \{0, \ldots, N\}$, depends on the practical choice of $\varphi_1, \ldots, \varphi_N$. The application of the classical Fourier basis (like [1, p. 139]) brings complications with averaged boundary values; thus the standard finite element technique, the wavelet analysis or other meshless approaches seem to be more efficient.

The solution $\alpha(\tau)$ of the system (5) can be analyzed with the help of real eigenvalues ω_n, $n \in \{0, \dots N\}$, obtained from the characteristic equation

$$\det(\lambda K - a^2 \omega_n \zeta M) = 0\,,$$

and of the corresponding real eigenvectors; alternatively this can be rewritten as

$$\lambda K V = a^2 \zeta M V \Omega\,,$$

where Ω is a diagonal square matrix of eigenvalues and V is a square matrix compound from column eigenvectors. All particular steps of this calculation can be found in [7]; the final result is

$$\alpha(\tau) = V \begin{bmatrix} \tau/(a\zeta) & & & \\ & \lambda/(a^2\zeta^2\omega_1)(1 - \exp(-a^2\zeta\omega_1\tau/\lambda)) & & \\ & & \cdots & \\ & & & \lambda/(a^2\zeta^2\omega_N)(1 - \exp(-a^2\zeta\omega_N\tau/\lambda)) \end{bmatrix} \quad (6)$$

$$\times V^{\mathrm{T}} M^{-1} (\beta_+ q_+ - \beta_- q_- + arg)\,.$$

Nevertheless, q_- and q_+ at all material interfaces are still undetermined. No external fluxes are allowed, thus only four unknown values q_-, q_+ at such interfaces occur. At the same interfaces four continuity conditions for T are available, consequently all needed q_-, q_+ can be evaluated formally from the corresponding regular system of 4 linear algebraic equations with 4 variables. Then we have

$$T_N(-l, t_s) = T_N(-l, t_{s-1}) + G_{s-}(\lambda, \zeta)\,, \qquad T_N(l, t_s) = T_N(l, t_{s-1}) + G_{s+}(\lambda, \zeta)$$

with two complicated functions G_{s-}, G_{s+} of two variables λ, ζ, coming from the insertion of (6) into (4) for $x = -l$ and $x = l$; the software code for the evaluation of G_{s-}, G_{s+} makes use of MAPLE. Now we are ready to specify the vague relations (3): the minimum of a function

$$\Phi(\lambda, \zeta) = \frac{1}{2} \sum_{\sigma \in \{-,+\}} \sum_{s=1}^{S} (T_N(\sigma l, t_s) - T_{s\sigma})^2$$

can be found with the help of the least squares method and (for a sufficiently good estimate of λ, ζ) of the Newton iterations, completed by an effective algorithm for the evaluation of the first and second partial derivatives of Φ needed in such iterations.

Material engineers in similar situations commonly use an "ad hoc" algorithm: i) set some rough estimate of λ and ζ; ii) by using some "black box" software like ANSYS, calculate the distribution of T in time, including that at the measured points; iii) if the differences between the measured and calculated values of T are large (which is decided from experience), choose another couple (λ, ζ) by using some heuristic technique (bi-sectioning, for example), and return to step ii), otherwise finish. The convergence of such approach is slow and doubtful; our semi-analytic

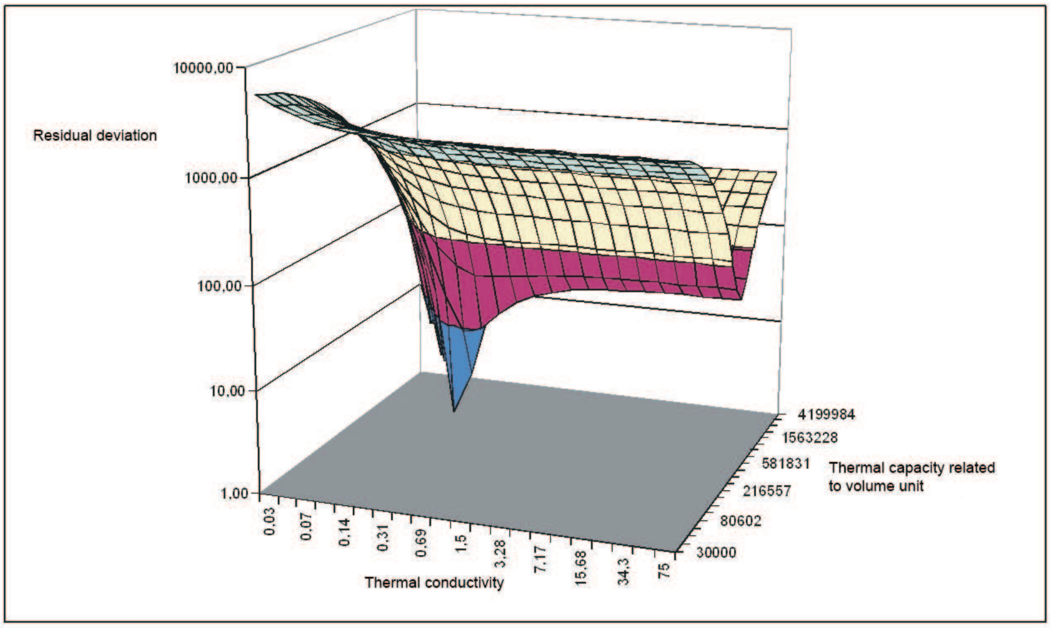

Fig. 3: *Characteristics λ, ζ, obtained by the minimization of $\Phi(\lambda, \zeta)$.*

method seems to be much more efficient. Nevertheless, the new revisions of commercial software packages such as ANSYS involve also certain support of the analysis of inverse problems, more advanced than the above criticized one. A typical distribution of $\Phi(\lambda, \zeta)$ for the (nearly) homogenized insulation layer, making use of the wood waste, is shown in Fig. 3.

4. Stochastic analysis

The analysis of uncertainties in measurements is an important part of the accreditation process of each technical laboratory. Most definitions of uncertainty in technical standards are rather vague, as "uncertainty is a parameter associated with the result of a measurement, that characterises the dispersion of the values that could reasonably be attributed to the measurand" in [3, p. 9]. However, to respect such requirements, the identification of thermal technical characteristics λ and ζ for new insulation materials should contain a deep analysis of uncertainty sources and components and their relation to random and systematic errors in measurements, concerning: i) the size of the material sample and the smoothness of its surface, ii) the correctness of setting of generated heat, iii) the correctness of temperature measurements from both sensors, iii) the preservation of assumed zero boundary fluxes, iv) the validity of homogenized isotropic constant values of the chararacteristics, v) the acceptability of physical, mathematical, and computational simplifications, vi) the numerical error analysis, etc.

For illustration, following [3, pp. 11, 24], we can reduce the analysis of uncertainty components to the analysis of standard deviations, assuming: i) the uncorrelated quantities r_s (adjusted values) and $T_{\sigma s}$ (measured values) with $s \in \{1, \ldots, S\}$,

$\sigma \in \{-,+\}$, ii) the normal (Gaussian) probability distribution (justified by the central limit theorem), and iii) the uncertainty w_r of all variables r_s and the uncertainty w_T of all variables T_s. Then the uncertainties w_λ and w_ζ of both material characteristics λ, ζ can be calculated as

$$w_\lambda = \sqrt{w_r^2 \sum_{s=1}^{S} (\partial\lambda/\partial r_s)^2 + w_T^2 \sum_{\sigma\in\{-,+\}} \sum_{s=1}^{S} (\partial\lambda/\partial T_{s\sigma})^2}\,,$$

$$w_\zeta = \sqrt{w_r^2 \sum_{s=1}^{S} (\partial\zeta/\partial r_s)^2 + w_T^2 \sum_{\sigma\in\{-,+\}} \sum_{s=1}^{S} (\partial\zeta/\partial T_{s\sigma})^2}\,.$$

A detailed study shows that evaluation of the above presented uncertainties can use the same algorithms as those in the Newton iteration process; this makes all computations relatively simple and inexpensive. More complicated formulae are needed in some other cases, e. g. in case of the uncertain thickness l.

Recently in [5] the approach presented in this paper has been applied to a room microclimate oriented study of the thermal behaviour of many new experimental materials for insulation layers in buildings. Unfortunately, especially the values of ζ (much more than those of λ) obtained both from the literature and from other experiments under similar conditions have a very large dispersion; thus (although the existence of solutions can be verified formally and the implemented software returns rather low values of $\Phi(\lambda,\zeta)$ – for illustration see Fig. 3 again) the validity of results, taking into account all potential sources of errors and inaccuracies, should be examined properly in the near future.

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